1968

Computer techniques for implementing linear control system design using algebraic methods.

Walker, John Andrew

Monterey, California. Naval Postgraduate School

COMPUTER TECHNIQUES FOR IMPLEMENTING LINEAR CONTROL SYSTEM DESIGN USING ALGEBRAIC METHODS

JOHN ANDREW WALKER

COMPUTER TECHNIQUES FOR IMPLEMENTING LINEAR CONTROL SYSTEM DESIGN USING ALGEBRAIC METHODS

by

John Andrew Walker Jr.
Lieutenant, United States Navy
B.S.M.E., University of Notre Dame, 1961

[redacted]

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
June 1968

ABSTRACT

Design of linear control systems is examined using algebraic methods. Previous work indicates that the system describing equations are nonlinear. Various combinations of feedback and cascade compensated systems are analyzed using velocity constant, bandwidth and complex root specifications. Methods for linearizing these system equations are demonstrated and examples are solved using specific digital computer programs. Feasibility of a general computer program is discussed.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

Page

TABLE OF SYMBOLS

s	complex variable of form $\sigma + j\omega$
ζ	damping ratio
w_n	undamped natural frequency
w_b	bandwidth frequency
$\phi_k(s)$	Mitrovic function
$G_o'(s)$	forward transfer function
$G_o(s)$	open loop transfer function
$G_c(s)$	closed loop transfer function
$H(s)$	feedback transfer function
K	forward system gain
K_t	tachometer gain
K_a	accelerometer gain
K_v	velocity constant

Acknowledgements

The author is indebted to Dr. G. J. Thaler of the Naval Postgraduate School for the guidance and direction provided during the preparation of this thesis.

I. Introduction

Design is a word of many meanings and forms. Generally, it means a method or procedure for determining the value of system parameters such that the system will respond in a prescribed manner. These methods are usually derived for synthesis in the time domain by applying modern state space techniques or in the complex frequency domain using root locus, Bode, Nyquist, or Nichol's plot.

All the methods have their advantages and disadvantages. Time domain analysis with state space allows the designer to examine the entire system at one time. He can only select one set of parameter values and investigate their effect on the system. The advent of digital computers greatly aided this process because the computer is easily adapted to a systematic trial and error design procedure.

The complex frequency plane methods are inherently trial and error. These methods are graphical and extremely time consuming in their application. Present day programming is gradually coming to the rescue but it has limitations in the amount of useful and legible information that can be displayed on a graph.

An outgrowth of the graphical methods was Mitrovic's method which was generalized by the Parameter Plane technique. This procedure transforms the analytical model of the system into a graphical representation. The benefit of this method is that it can handle multiparameters.

From the parameter plane technique system design has returned to the analytical world and developed a multi-parameter procedure called algebraic methods. Algebraic methods reduces the system differential equations and specifications prescribing the desired response into a set of simultaneous algebraic equations. The simplicity of the method is that it finds the parameter values that collectively provide the required system response. The method has developed over the years but its implementation on computers has not been explored. It is the intention of this thesis to investigate this area.

II. Basic Theory

The backbone of the algebraic methods is expressing the systems characteristic equation as a polynomial of the form

$$\sum_{K=0}^{n} a_K S^K = 0 \qquad (2\text{-}1)$$

where s is the complex variable and is defined as

$$S = -\zeta \omega_n + j\omega_n\sqrt{1-\zeta^2} \qquad (2\text{-}2)$$

and the a_k's are real coefficients which are functions of the system parameters. Two independent equations in a_k, ζ, and w_n are obtained if equation (2-2) is substituted in equation (2-1) and the real and imaginary parts set equal to zero. Performing additional manipulation and introducing a new variable (1), (2), we obtain

$$\sum_{K=0}^{n} a_K \omega_n^K \phi_{K-1}(\zeta) = 0 \qquad (2\text{-}3)$$

$$\sum_{K=0}^{n} a_K \omega_n^K \phi_K(\zeta) = 0 \qquad (2\text{-}4)$$

where

$$\phi_K(\zeta) = -\left[2\zeta\phi_{K-1}(\zeta) + \phi_{K-2}(\zeta)\right] \qquad (2\text{-}5)$$

with

$$\phi_0(\zeta) = 0 \;, \quad \phi_1(\zeta) = -1 \quad \text{AND} \quad \phi_{-K}(\zeta) = -\phi_K(\zeta) \qquad (2\text{-}6)$$

Other values of $\phi_K(\zeta)$ for various values of ζ are tabulated in Appendix I. Observe that for a real root $S = -\sigma$ and substituting in equation (2-1) gives

$$\sum_{K=0}^{n} (-1)^K a_K \sigma^K = 0 \qquad (2\text{-}7)$$

Previous work provided graphical methods for portraying root locations as a function of any two system parameters. (2), (3), (4), (5). If we are willing to abandon graphical analysis then the equations can include any number of unknown parameters. Proceed algebraically by substituting root locations in the basic equations (2-3) and (2-4) giving two equations as functions of all the unknowns. Requirement for solution is the number of equations equal the number of unknowns. One obtains additional equations by one of two philosophies; first, substitution of additional root locations into the basic equations; second, derivation of additional algebraic relationships from system performance specifications. Determination and solution of the proper number of equations results in a simultaneous set of parameter values that satisfy stated requirements. Naturally, if the parameter values are not satisfactory the performance specifications must be altered or the additional equations generated changed and a new solution affected.

Inherently these equations provide a set of nonlinear simultaneous equations to be solved. The nonlinearities are generated from the system configuration or conversion of the performance specifications into algebraic relationships. The purpose of this thesis is to develop methods of solving these nonlinear equations. Basic approach will be that of linearization by substitution or manipulating the form of system components to facilitate linearization. When linearization fails it is hoped the system will lend itself to one of the standard analytical methods for solving linear simultaneous equations (6).

III. Third Order Development

The basic system general block diagram is shown in Figure 3-1. In this section we will use a type one, third order plant to demonstrate the different solution techniques. Naturally, it is required that some type of compensation be employed, namely, feedback, cascade and feedback-cascade combination, so that the system satisfies the specifications. The third order system allows for development of the method while keeping the equation length reasonable. As needed reference will be made to higher order systems and some higher order examples will be solved.

3.1 Feedback Compensation

Consideration will first be given to tachometer feedback, Figure 3-2, where the system gain (K) and the tachometer gain (K_t) are unknown. Solution for the unknowns requires two simultaneous equations. Since the most common system specification is a pair of dominant complex roots, equation (2-3) and (2-4) will be implemented.

The system characteristic equation is

$$S^3 + (P1 + P2) S^2 + (P1 P2 + K K_t) S + K = 0 \qquad (3\text{-}1)$$

from which the a_k coefficients and the given ζ and ω_n are substituted in the root equations giving

$$K \phi_{-1}(\zeta) + (P1\,P2 + K K_t)\,\omega_n \phi_0(\zeta) + (P1 + P2)\,\omega_n^2 \phi_1(\zeta) + \omega_n^3 \phi_2(\zeta) = 0 \qquad (3\text{-}2)$$

$$K \phi_0(\zeta) + (P1 P2 + K K_t)\,\omega_n \phi_1(\zeta) + (P1 + P2)\,\omega_n^2 \phi_2(\zeta) + \omega_n^3 \phi_3(\zeta) = 0 \qquad (3\text{-}3)$$

It can be seen these equations are nonlinear due to the KK_t term. Linear equations are obtained by making the following transformation of variables

$$\begin{aligned} X &= K \\ Y &= KK_t \end{aligned} \tag{3-4}$$

After making the substitution and rearranging we get the following matrix form

$$\begin{bmatrix} \phi_{-1}(S) & W_n \phi_0(S) \\ \phi_0(S) & W_n \phi_1(S) \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} -P1P2\, W_n \phi_0(S) - (P1+P2) W_n^2 \phi_1(S) - W_n^3 \phi_2(S) \\ -P1P2\, W_n \phi_1(S) - (P1+P2) W_n^2 \phi_2(S) - W_n^3 \phi_3(S) \end{bmatrix} \tag{3-5}$$

Linear equations are easily solved by digital computers with programs using matrix algebra for implementing the standard linear equation solving techniques. It is a simple operation to inverse transform the linearizing substitution and obtain the parameter values

$$K = X$$

and (3-6)

$$K_t = Y/K$$

Observe that for this specific case the matrix equation degenerates to give

$$X = \frac{-(P1+P2) W_n^2 \phi_1(S) - W_n^3 \phi_2(S)}{\phi_{-1}(S)} \tag{3-7}$$

and

$$Y = \frac{-P1P2\, \phi_1(S) - (P1+P2) W_n \phi_2(S) - W_n^2 \phi_3(S)}{\phi_1(S)} \tag{3-8}$$

because $\phi_0(S) = 0$

Another possible method of feedback is acceleration feedback, Figure 3-3, which is very similar to tachometer feedback but affects the second order coefficient of the characteristic equation instead of the first order coefficient. This type of compensation generates a system with two unknowns, therefore, requiring two equations which we will obtain from the root specifications. The system characteristic equation is

$$S^3+(P1+P2+KK_a)S^2+P1P2S+K=0 \tag{3-9}$$

Appropriate substitution in the root equations yields

$$K\phi_{-1}(S)+P1P2\omega_n\phi_0(S)+(P1+P2+KK_a)\omega_n^2\phi_1(S)+\omega_n^3\phi_2(S)=0 \tag{3-10}$$

and

$$K\phi_0(S)+P1P2\omega_n\phi_1(S)+(P1+P2+KK_a)\omega_n^2\phi_2(S)+\omega_n^3\phi_3(S)=0 \tag{3-11}$$

As in the tachometer feedback case the following linearizing transformation of variables can be made

$$\begin{aligned} X&=K \\ Y&=KK_a \end{aligned} \tag{3-12}$$

Making the substitution and rearranging gives the matrix simultaneous equations

$$\begin{bmatrix} \phi_{-1}(S) & \omega_n^2\phi_1(S) \\ \phi_0(S) & \omega_n^2\phi_2(S) \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} -P1P2\omega_n\phi_0(S)-(P1+P2)\omega_n^2\phi_1(S)-\omega_n^3\phi_2(S) \\ -P1P2\omega_n\phi_1(S)-(P1+P2)\omega_n^2\phi_2(S)-\omega_n^3\phi_3(S) \end{bmatrix} \tag{3-13}$$

For this type of feedback we do not get the degeneracy due to the fortunate location of $\phi_0(S)$, but it is seen the matrix equations for tachometer and acceleration feedback are the same except for the second column of the coefficient matrix. Therefore, we can expect that when both types of feedback are used simultaneously the matrix equation will be very similiar to (3-5) and (3-12). At this point we will state the results if a fourth order system is used. The matrix equation is

$$\begin{bmatrix} \phi_{-1}(S) & \omega_n^2\phi_1(S) \\ \phi_0(S) & \omega_n^2\phi_2(S) \end{bmatrix} \begin{bmatrix} K \\ KK_a \end{bmatrix} = \begin{bmatrix} -P3P1P2\omega_n\phi_0(S)-(P1P2+P1P3+P2P3)\omega_n^2\phi_1(S)-(P1+P2+P3)\omega_n^3\phi_2(S) \\ -\omega_n^4\phi_3(S) \\ -P3P1P2\omega_n\phi_1(S)-(P1P2+P1P3+P2P3)\omega_n^2\phi_2-(P1+P2+P3)\omega^3\phi_3(S) \\ -\omega_n^4\phi_4(S) \end{bmatrix} \tag{3-14}$$

From this it is seen that the form is the same as the third order case and all the increase in system order did was add a term to the right hand side.

Let us now combine the types of feedback, Figure 3-4, and solve the problem. Immediately it is seen that before we can proceed we have to develop another equation besides the root equations. A frequently used specification is error constants which has different meaning depending on the type of system. For the type one system this means we can use the velocity constant defined as

$$K_v = \lim_{s\to 0} SG_o(S) \tag{3-15}$$

with the steady state error for a ramp input, $R(s) = 1/s^2$

$$e_{ss} = \lim_{s \to 0} \frac{SR(S)}{1+G_0(S)} = \lim_{s \to 0} \frac{1}{SG_0(S)} = \frac{1}{K_V} \qquad (3\text{-}16)$$

The characteristic equation for the system is

$$S^3 + (P1+P2+KK_a)S^2 + (P1P2+KK_t)S + K = 0 \qquad (3\text{-}17)$$

and the forward transfer function is

$$G_0(S) = \frac{K}{S[(S+P1)(S+P2)+KK_t+KK_aS]} \qquad (3\text{-}18)$$

Applying the root equations and the velocity coefficient equation we obtain the following set of simultaneous non-linear equations in K, KK_t, and KK_a

$$K_V P1P2 + K_V KK_t - K = 0 \qquad (3\text{-}19)$$

$$\phi_{-1}(S)K + (P1P2+KK_t)\omega_n\phi_0(S) + (P1+P2+KK_a)\omega_n^2\phi_1(S) + \omega_n^3\phi_2(S) = 0 \qquad (3\text{-}20)$$

$$\phi_0(S)K + (P1P2+KK_t)\omega_n\phi_1(S) + (P1+P2+KK_a)\omega_n^2\phi_2(S) + \omega_n^3\phi_3(S) = 0 \qquad (3\text{-}21)$$

As in the separate type of feedback compensation we can apply the same transformation of variables to linearize the equations

$$\begin{aligned} X &= K \\ Y &= KK_t \\ Z &= KK_a \end{aligned} \qquad (3\text{-}22)$$

Making the substitution and rearranging we obtain the matrix equation

$$\begin{bmatrix} -1 & K_V & 0 \\ \phi_{-1}(S) & \omega_n\phi_0(S) & \omega_n^2\phi_1(S) \\ \phi_0(S) & \omega_n\phi_1(S) & \omega_n^2\phi_2(S) \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} -K_V P1P2 \\ -P1P2\omega_n\phi_0(S) - (P1+P2)\omega_n^2\phi_1(S) - \omega_n^3\phi_2(S) \\ -P1P2\omega_n\phi_1(S) - (P1+P2)\omega_n^2\phi_2(S) - \omega_n^3\phi_3(S) \end{bmatrix} \qquad (3\text{-}23)$$

As predicted the above result is very similar to the feedback cases examined separately. The coefficient matrix second column is from the tachometer equation (3-5) and the acceleration equation (3-13). Similarly, the right side is not changed at all. From this we conclude the type of compensation for the root location specification affects terms in the coefficient matrix while the right hand side is affected only by the system order.

A possible specification for generating an extra equation is bandwidth. The conventional definition of bandwidth will be used, that is, the frequency at which the magnitude of the output is one half the input magnitude for the closed loop transmission with $S = jW_b$. Because the magnitude involves squaring the transfer function we can expect to gain a nonlinear equation of a rather difficult form. For feedback compensation, it is found the problem is not extremely difficult.

Consider the tachometer feedback, Figure 3-2, where the open loop transfer function is

$$G_o(S) = \frac{K}{S(S+P1)(S+P2) + KK_tS} \tag{3-24}$$

and the closed loop transfer function is

$$G_c(S) = \frac{K}{S(S+P1)(S+P2) + KK_tS + K} \tag{3-25}$$

to which we apply the bandwidth requirement giving

$$\frac{1}{2} = \frac{K^2}{-W_b^2\left[P1P2 - W_b^2 + KK_t\right]^2 + \left[K - W_b^2(P1+P2)\right]^2} \tag{3-26}$$

To simplify the expression make the substitution

$$A = -\omega_b^2 (P1 + P2)$$

and

$$B = P1P2 - \omega_b^2 \qquad (3\text{-}27)$$

whereby clearing gives

$$-K^2 + 2AK - 2B\omega_b^2 KK_t + A^2 - \omega_b^2 B^2 - \omega_b^2 K^2 K_t^2 = 0 \qquad (3\text{-}28)$$

Before proceding any further let us examine the second equation required for solution. Select the velocity constant specification because a complex root specification generates two equations and it is not our intention to deal with over determined systems, hence using

$$K_V = \lim_{S \to 0} S\left[\frac{K}{S(S+P1)(S+P2) + KK_t S}\right]$$

gives

$$K_V KK_t - K = -K_V P1P2 \qquad (3\text{-}29)$$

Examination of equations (3-28) and (3-29) shows an extreme nonlinearity. Fortunately, if we consider KK_t as an unknown we can use the elimination method to reduce the system to a quadratic equation in forward gain K. Solving equation (3-29) for KK_t

$$KK_t = (K - K_V P1P2)/K_V \qquad (3\text{-}30)$$

and substituting this value in equation (3-28) gives

$$\left[-1.0 - \omega_b^2/K_V^2\right] K^2 + \left[2A - 2\omega_b^2/K_V + 2\omega_b^2 P1P2/K_V\right] K$$
$$+ \left[2\omega_b^2 BP1P2 + A^2 - \omega_b^2 B^2 - \omega_b^2 (P1P2)^2\right] = 0 \qquad (3\text{-}31)$$

This is easily solved for the system gain K yielding two values which can be substituted in equation (3-30) to give two values of K_t

$$K_{t_{1,2}} = 1/K_v - P1P2/K_{1,2} \tag{3-32}$$

At this point the designer has a choice of which set of gains to use for the solution. Usually it is not a difficult choice because one of the gain values is negative. Situations where both gains are positive values the designer can pick the values that suit his purpose.

Now we'll shift our attention to the acceleration feedback, Figure 3-3. From previous expressions it can be expected the feedback gain in the second order term will help the solution. As in the tachometer case we will use bandwidth and velocity constant specifications. The velocity coefficient is

$$K_v = K/P1P2$$

giving one equation

$$K = K_v P1P2 \tag{3-33}$$

The bandwidth specification gives

$$\frac{1}{2} = \frac{K^2}{-\omega_b^2 A^2 + [-\omega_b^2 K K_a + K - B]^2} \tag{3-34}$$

where $A = P1P2 - \omega_b^2$ AND $B = \omega_b^2(P1+P2)$

Rearrangement and substitution for K from (3-33) gives a a quadratic in K_a.

$$\omega_b^4 (P1P2KV)^2 Ka^2 + (-2\omega_b^2 (P1P2Kv)^2 + 2\omega_b^2 BP1P2Kv)Ka$$

$$+ [B^2 - \omega_b^2 A^2 - 2BP1P2Kv - (P1P2Kv)^2] = 0 \qquad (3\text{-}35)$$

Again this gives two values for the acceleration and forward gains. As in the tachometer case the choice is left to the designer but usually one of the choices will have an undesirable negative gain.

3.2 Cascade Compensation

Cascade compensation is a widely used method of compensation. In general, it is commonly referred to as a lead, lag, or lead-lag filter. In this development we will not prespecify the type of filter or a criteria for realizability. The decision of realizability is left to the system or filter designer.

The first case examined is the single section compensator, Figure 3-5. In keeping with the major requirement we need three equations to solve for the unknown system gain (K), filter zero(Z), and the filter pole(P). If given a complex root specification and a velocity constant we can generate the required equations. A possible unknown is eliminated by considering any gain associated with the filter to be included in the overall system gain.

The system characteristic equation is

$$s^4 + (P1+P2+P)s^3 + (P(P1+P2)+P1P2)s^2 + (P1P2P+K)s + KZ = 0 \qquad (3\text{-}36)$$

indicating one nonlinearity, the KZ term. The velocity constant specification yields

$$k_v = \left. \frac{K(s+z)}{(s+P)(s+P1)(s+P2)} \right|_{s=0} = \frac{Kz}{P1P2P} \tag{3-37}$$

which has the same nonlinear term as the characteristic equation, making the now familiar transformation of variables

$$X = Kz \tag{3-38}$$

provides linearity with the three unknowns P, K, X which after affecting the root equations gives the matrix equation

$$\begin{bmatrix} 0 & -K_v P1P2 & 1 \\ \omega_n \phi_0 & P1P2\,\omega_n \phi_0 + (P1+P2)\omega_n^2 \phi_1 + \omega_n^3 \phi_2 & \phi_1 \\ \omega_n \phi_1 & P1P2\,\omega_n \phi_1 + (P1+P2)\omega_n^2 \phi_2 + \omega_n^3 \phi_3 & \phi_0 \end{bmatrix} \begin{bmatrix} K \\ P \\ X \end{bmatrix} = \begin{bmatrix} 0 \\ -P1P2\,\omega_n^2 \phi_1 - (P1+P2)\omega_n^3 \phi_2 - \omega_n^4 \phi_3 \\ -P1P2\,\omega_n^2 \phi_2 - (P1+P2)\omega_n^3 \phi_3 - \omega_n^4 \phi_4 \end{bmatrix} \tag{3-39}$$

After solution we once again inverse transform the variable for the filter pole

$$z = X/K \tag{3-40}$$

Unlike the feedback system a change in the system order affects not only the right side of the matrix equation but, also, the left hand coefficient matrix. The reason for this effect can be seen if the reduction of the system to a forward transfer function with unity feedback is examined. The filter pole multiplies all poles of the basic plant, thus inserting the filter pole in all the characteristic coefficients except a_n and a_o.

The next logical consideration is a multisection cascaded filter. To demonstrate the best analysis form we will use the double section filter, Figure 3-6. The addition of each filter section adds two more unknowns, the section zero and pole, which requires two extra equations per section added. At the risk of completely specifying the system roots the easiest specification to use giving two equations is a root specification, otherwise other specifications will have to be translated into algebraic equations. The root specification can be very helpful since if not given explicitly it can be selected such that it does not effect dominance of any specified roots.

Before proceding further, to ease the notation, multiply the plant denominator giving

$$S^3 + AS^2 + BS \tag{3-41}$$

where

$$\begin{aligned} A &= P_1' + P_2' \\ B &= P_1' P_2' \end{aligned} \tag{3-42}$$

Reducing the overall system by normal methods, we achieve the following characteristic equation

$$\begin{aligned} &S^5 + (A + P1 + P2)S^4 + (B + A(P1+P2) + P1P2)S^3 + (B(P1+P2) \\ &+ A(P1P2) + K)S^2 + (BP1P2 + KZ1 + KZ2)S + KZ1Z2 = 0 \end{aligned} \tag{3-43}$$

At this point, note the transformation of the unknowns into the seven groupings P1, P2, P1P2, KZ1, KZ2, K, and KZ1Z2. This double section may not be that unwieldy to solve but the situation gets worse as the sections increase. A simple manipulation can alleviate this undesirable condition.

Conversion of the filter to polynomial form vice factored form prevents the generation of undesirable nonlinear terms. Granted some remain but they can be handled by appropriate variable transformation. The polynomial form of the filter is

$$\frac{s^2 + XS + Y}{s^2 + WS + Z} \tag{3-44}$$

where

$$\begin{aligned} X &= Z_1 + Z_2 \\ Y &= Z_1 Z_2 \\ W &= P_1 + P_2 \\ Z &= P_1 P_2 \end{aligned} \tag{3-45}$$

Using this filter form in the system reduction we achieve a new and simpler characteristic equation

$$\begin{aligned} &s^5 + (A+W)s^4 + (B+AW+Z)s^3 + (BW+AZ+K)s^2 \\ &+ (BZ+KX)s + KY = 0 \end{aligned} \tag{3-46}$$

The nonlinear terms can be linearized by

$$\begin{aligned} R &= KY \\ S &= KX \end{aligned} \tag{3-47}$$

The application of the root equations twice and the velocity constant

$$K_v = \lim_{s \to 0} s \frac{K(s^2 + XS + Y)}{(s^2 + WS + Z)(s^3 + As^2 + BS)} = \frac{KY}{BZ} \tag{3-48}$$

yields the matrix simultaneous equation form

$$[A]\begin{bmatrix} R \\ Z \\ S \\ K \\ W \end{bmatrix} = [B] \tag{3-49}$$

where for root specification one the coefficients

$$
\begin{aligned}
A(1,1) &= \phi_1(S) \\
A(1,2) &= A\omega_n^2\phi_1(S) + \omega_n^3\phi_2(S) + B\omega_n\phi_0(S) \\
A(1,3) &= \omega_n\phi_0(S) \\
A(1,4) &= \omega_n^2\phi_1(S) \\
A(1,5) &= B\omega_n^2\phi_1(S) + A\omega_n^3\phi_2(S) + \omega_n^4\phi_3(S) \\
A(2,1) &= \phi_0(S) \\
A(2,2) &= B\omega_n\phi_1(S) + A\omega_n^2\phi_2(S) + \omega_n^3\phi_3(S) \\
A(2,3) &= \omega_n\phi_1(S) \\
A(2,4) &= \omega_n^2\phi_2(S) \\
A(2,5) &= B\omega_n^2\phi_2(S) + A\omega_n^3\phi_3(S) + \omega_n^4\phi_4(S) \\
B(1) &= -B\omega_n^2\phi_2(S) - A\omega_n^4\phi_3(S) - \omega_n^5\phi_4(S) \\
B(2) &= -B\omega_n^3\phi_3(S) - A\omega_n^4\phi_4(S) - \omega_n^5\phi_5(S)
\end{aligned}
\tag{3-50}
$$

are functions of zeta one and omega one. For root specification two the third and fourth rows have the same form as rows one and two, ie,

$$
\begin{aligned}
A(3,i) &= A(1,i) \\
A(4,i) &= A(2,i)
\end{aligned}
\tag{3-51}
$$

and

$$
\begin{aligned}
B(3) &= B(1) \\
B(4) &= B(2)
\end{aligned}
$$

where A(3,i), A(4,i), B(3) and B(4) are functions of zeta two and omega two. The remaining row is

$$A(5,1) = -1.0$$
$$A(5,2) = K_v B \qquad (3\text{-}52)$$
$$A(5,i) = B(5) = 0.0 \qquad i = 3,4,5$$

From this development we can see the polynomial form filter provides the transformation of variables from Z_i to Ka_{i-1} for i = 1, ...,n. After solution of the linear equations the filter zeros and poles can be found by factoring the numerator and denominator.

3.3 Combined Compensation

Having considered feedback and cascade compensation individually we will now examine the combination of both types of compensation, Figure 3-1. Attention is given first to tachometer-cascade compensation, Figure 3-8, which indicates nonlinearity may be a problem. In this section the system gain is assumed known including the filter gain. Naturally, the three required equations are from velocity constant and root specification.

The characteristic equation is

$$s^4 + (P1+P2+P)s^3 + (PP1+PP2+P1P2+KK_t)s^2$$
$$+ (P1P2P + KK_tZ + K)s + KZ = 0 \qquad (3\text{-}53)$$

from which we apply the root equations and obtain the nonlinear matrix equation

$$\begin{bmatrix} A & B & C & O \\ E & O & F & D \\ R & S & O & H \end{bmatrix} \begin{bmatrix} P \\ K_t z \\ z \\ K_t \end{bmatrix} = \begin{bmatrix} O \\ G \\ T \end{bmatrix} \tag{3-54}$$

where

$$\begin{aligned}
A &= K_v P1 P2 \\
B &= K K_v \\
C &= -K \\
D &= \omega_n^2 \phi_1(s) K \\
E &= (P1+P2)\omega_n^2 \phi_1(s) + \omega_n^3 \phi_2(s) \\
F &= K \phi_1(s) \\
G &= -P1P2\,\omega_n^2 \phi_1(s) - (P1+P2)\omega_n^3 \phi_2(s) - \omega_n^4 \phi_3(s) \\
H &= \omega_n^2 \phi_2(s) K \\
R &= P1P2\,\omega_n \phi_1(s) + (P1+P2)\omega_n^2 \phi_2(s) + \omega_n^3 \phi_3(s) \\
S &= K \omega_n \phi_1(s) \\
T &= -K\omega_n \phi_1(s) - P1P2\,\omega_n^2 \phi_2(s) - (P1+P2)\omega_n^3 \phi_3(s) - \omega_n^4 \phi_4(s)
\end{aligned} \tag{3-55}$$

Similar to the bandwidth situation this case lends itself to the elimination method of solution such that equation (3-54) reduces to a quadratic equation in tachometer gain K_t from which the compensator pole and zero may be found. As previously the two answers give a choice which should operate under the same restrictions previously discussed.

Finally, we shall consider the combination of acceleration feedback-cascade compensation. Experience indicates the acceleration gain will simplify the equations, but for this case we find the compensator zero prevents this complete simplification. Using the root and velocity constant specifications with the characteristic equation

$$S^4 + (P1+P2+P+KK_a)S^3 + (P1P2+PP1+PP2+KK_a z)S^2 + (P1P2P+K)S + Kz = 0 \tag{3-56}$$

we obtain the nonlinear matrix equation

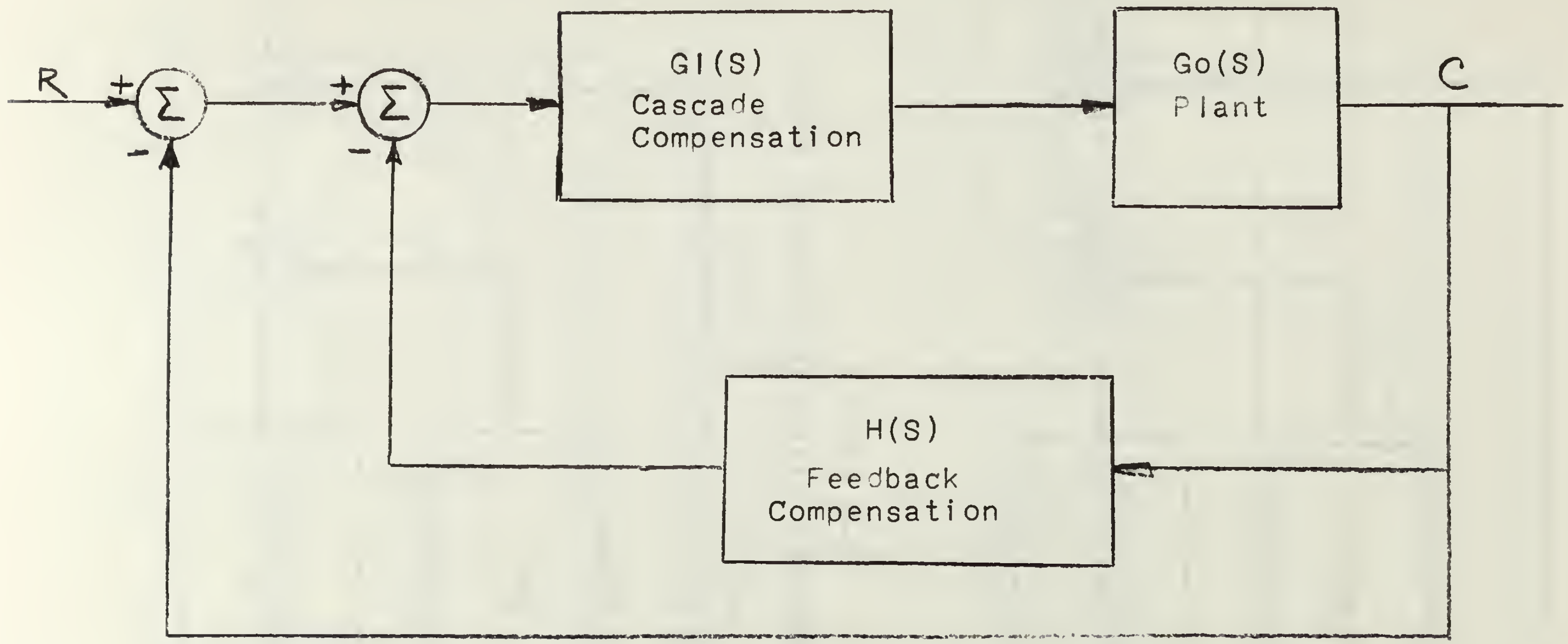
$$\begin{bmatrix} A & B & 0 & 0 \\ C & D & E & F \\ R & 0 & S & T \end{bmatrix} \begin{bmatrix} P \\ z \\ K_a \\ K_a z \end{bmatrix} = \begin{bmatrix} 0 \\ G \\ H \end{bmatrix} \tag{3-57}$$

where

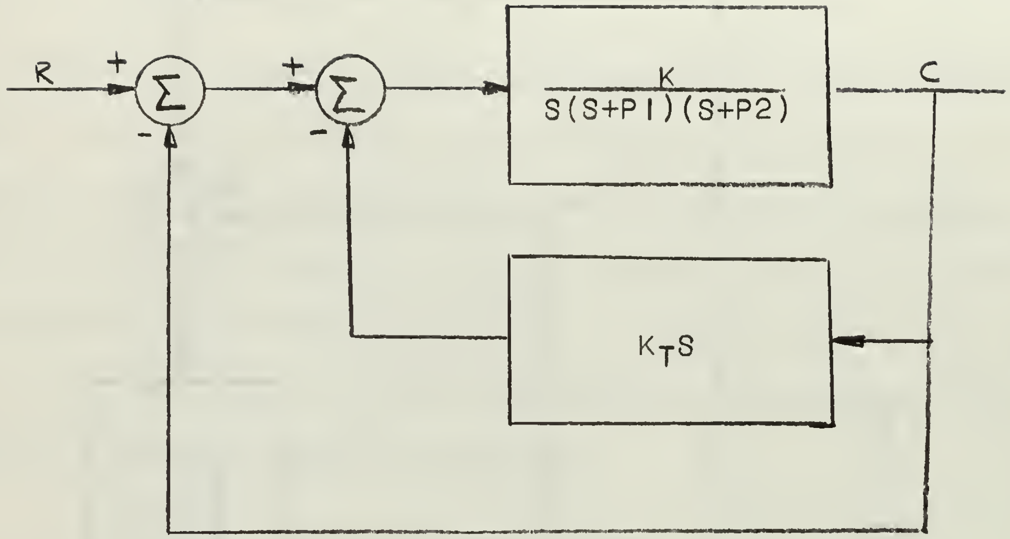
$$\begin{aligned}
A &= K_v P1P2 \\
B &= -K \\
C &= (P1+P2)\omega_n^2\phi_1(\zeta) + \omega_n^3\phi_2(\zeta) \\
D &= -K\phi_1(\zeta) \\
E &= K\omega_n^2\phi_1(\zeta) \\
F &= K\omega_n^3\phi_2(\zeta) \\
G &= -P1P2\,\omega_n^2\phi_1(\zeta) - (P1+P2)\omega_n^3\phi_2(\zeta) - \omega_n^4\phi_3(\zeta) \\
H &= -K\omega_n\phi_1(\zeta) - P1P2\,\omega_n^2\phi_2(\zeta) - (P1+P2)\omega_n^3\phi_3(\zeta) - \omega_n^4\phi_4(\zeta) \\
R &= \omega_n^3\phi_3(\zeta) + P1P2\,\omega_n\phi_1(\zeta) + (P1+P2)\omega_n^2\phi_2(\zeta) \\
S &= K\omega_n^2\phi_2(\zeta) \\
T &= K\omega_n^3\phi_3(\zeta)
\end{aligned} \tag{3-58}$$

The system of equations can be reduced by elimination (like the tachometer case) to a quadratic in acceleration gain K_a from which the compensator pole and zero can be found.

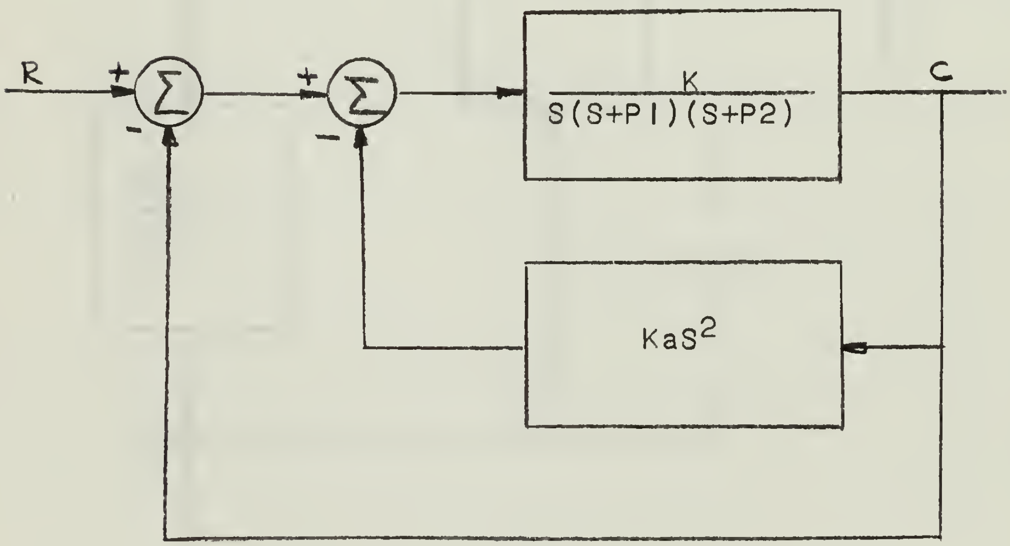
In this section we see the simultaneous equations easily lend themselves to solution by elimination but appear to be close to linear. This is true if a fourth specification is given that doesn't create nonlinearities (like bandwidth). The linearization method used throughout this thesis can be used for the K_tZ or K_aZ term. The situation is somewhat easier if a multisection filter is used. There is a nonlinear product for each section of filter. Therefore, if the number of filter sections is even then specify an additional k/2 complex roots, linearize and solve.



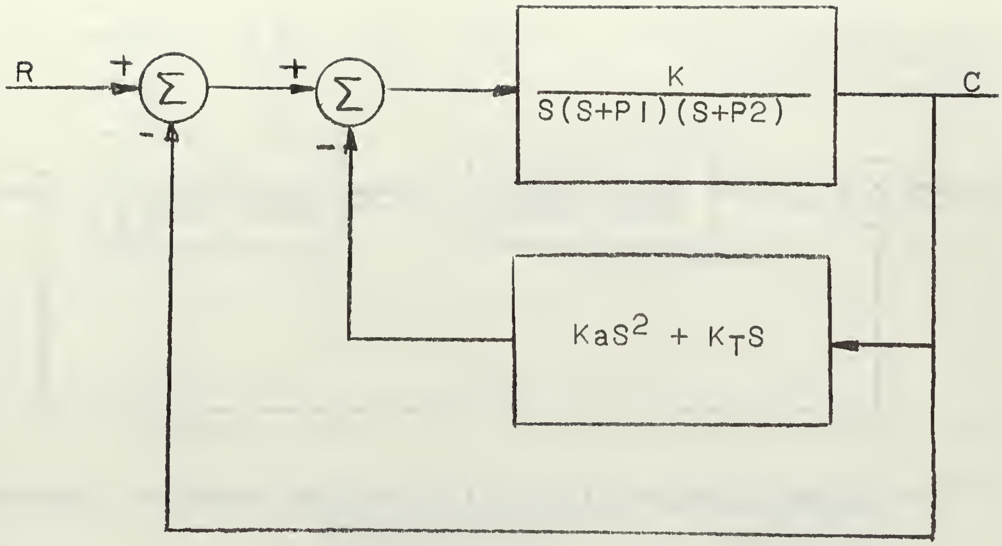
Basic System Block Diagram
Figure 3-1



Third Order System With Tachometer Feedback
Figure 3-2

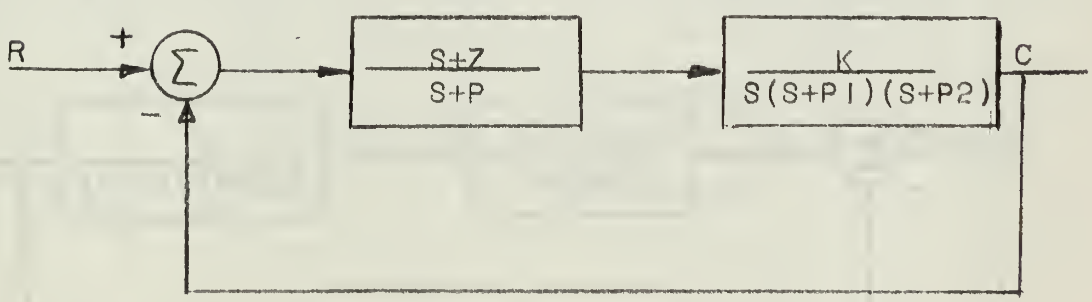


Third Order System With Acceleration Feedback
Figure 3-3



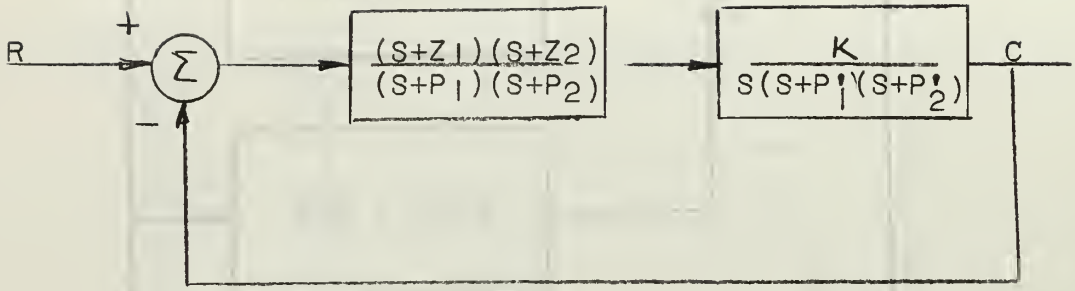
Third Order System With Tachometer And Acceleration Feedback

Figure 3-4



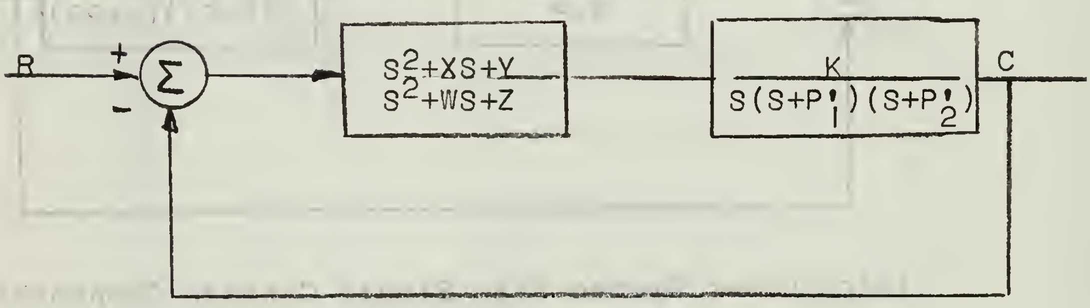
Third Order System With Single Cascade Compensation

Figure 3-5



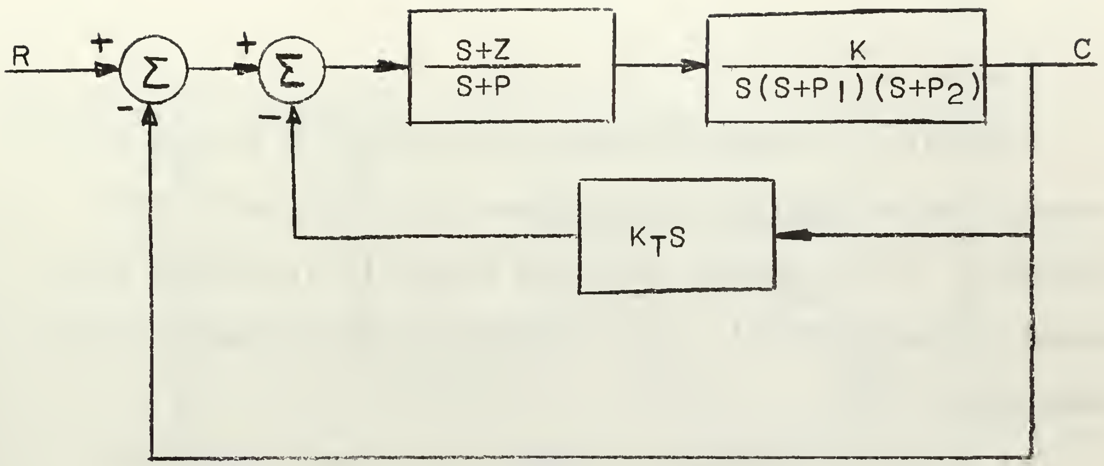
Third Order System With Multisection Cascade Compensation
Factored Form

Figure 3-6



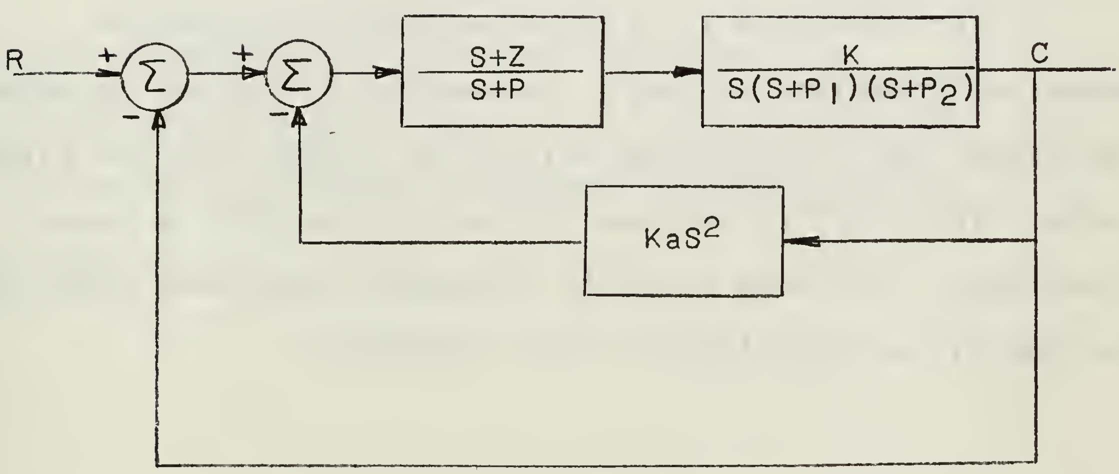
Third Order System With Multisection Cascade Compensation
Polynomial Form

Figure 3-7



Third Order System With Cascade
And Tachometer Feedback Compensation

Figure 3-8



Third Order System With Cascade
And Acceleration Feedback Compensation

Figure 3-9

IV. Examples

Examples in this section were solved on an IBM 360 computer using subroutine programs furnished with the computer. The programs used are specific in nature and appear in Appendix II. The following points apply to all examples:

a) negative feedback is desired so that negative feedback constants are not acceptable since this implies positive feedback.

b) when a cascade pole or zero is found positive sign indicates left half plane and negative sign indicates right half plane.

c) for bandwidth and feedback-cascade combination examples, there are no logic statements in programs to select the proper set of parameter values for determining the system roots. This part of the program was written with a priori knowledge. For roots the sign convention mentioned above does not apply, use the standard sign convention.

Example 4.1

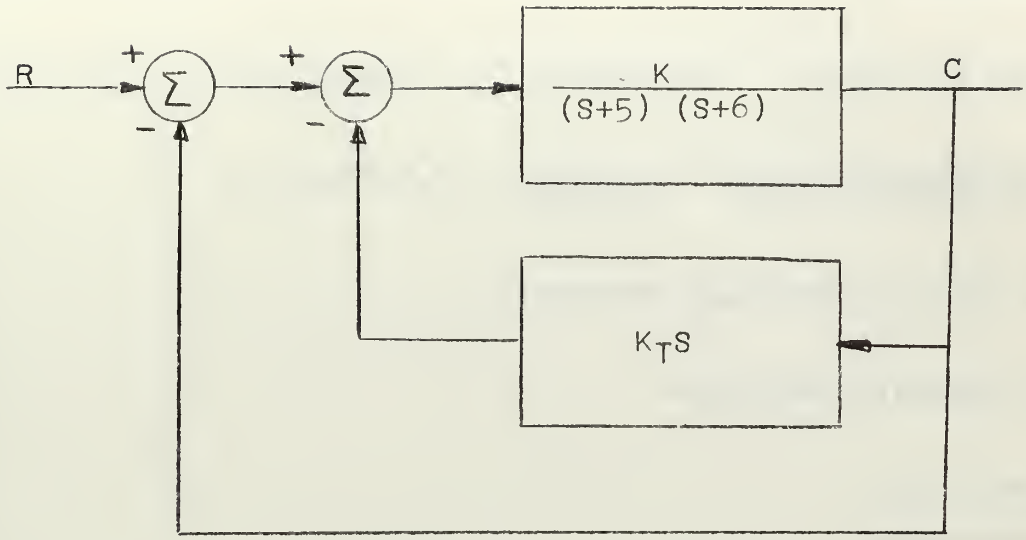


Figure 4.1

Using feedback compensation design a system with open loop transfer function in Figure 4.1.

Root Specification: Zeta = 0.5, Natural Frequency = 10.0

No Velocity Constant or Bandwidth Specification

```
INPUT DATA
**********


POLES ARE ZERO,    0.50000D 01,    0.60000D 01


ROOT SPECIFICATIONS ARE ZETA=    0.50000D 00,
NATURAL FREQUENCY=    0.10000D 02


 NO VELOCITY CONSTANT SPECIFIED


NO BANDWIDTH SPECIFIED


OUTPUT DATA
***********


FORWARD GAIN

    0.10000E 03


TACHOMETER GAIN

    0.80000E 00


COEFFICIENTS OF CHARACTERISTIC EQUATION
(ASCENDING ORDER)

    0.10000E 03    0.11000E 03    0.11000E 02    0.10000E 01

 THE SYSTEM ROOTS ARE



 REAL PART


   -0.10000E 01   -0.50000E 01   -0.50000E 01


 IMAGINARY PART


    0.0            -0.86603E 01    0.86603E 01
```

Example 4.2

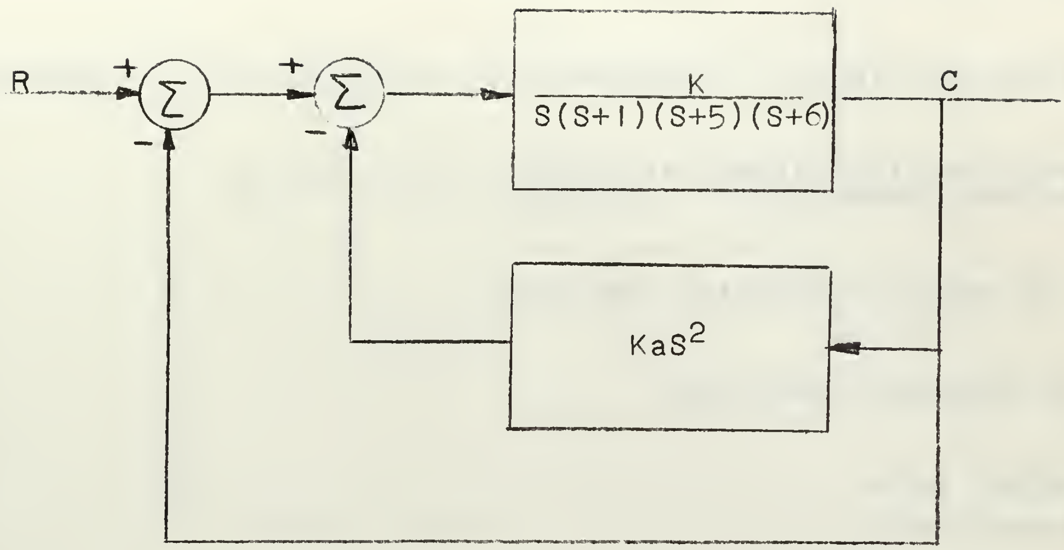


Figure 4.2

Design an acceleration feedback system with the open loop transfer function shown in Figure 4.2.

Root specification: Zeta = 0.5, natural frequency = 11.0

No velocity constant and bandwidth specification.

This example indicates an unattainable solution since solution gives negative K and Ka and a right half plane root.

```
INPUT DATA
**********

POLES ARE ZERO,    0.10000D 01,   0.50000D 01,   0.60000D 01

ROOT SPECIFICATIONS ARE ZETA=    0.50000D 00,
NATURAL FREQUENCY=    0.11000D 02

 NO VELOCITY CONSTANT SPECIFIED

NO BANDWIDTH SPECIFIED

OUTPUT DATA
***********

FORWARD GAIN

   -0.10010E 04

ACCELERATION GAIN

   -0.82645E-01

COEFFICIENTS OF CHARACTERISTIC EQUATION
(ASCENDING ORDER)

   -0.10010E 04    0.30000E 02    0.12373E 03    0.12000E 02
    0.10000E 01

 THE SYSTEM ROOTS ARE

 REAL PART

   -0.34194E 01    0.24194E 01   -0.55000E 01   -0.55000E 01

 IMAGINARY PART

    0.0            0.0           -0.95263E 01    0.95263E 01
```

Example 4.3

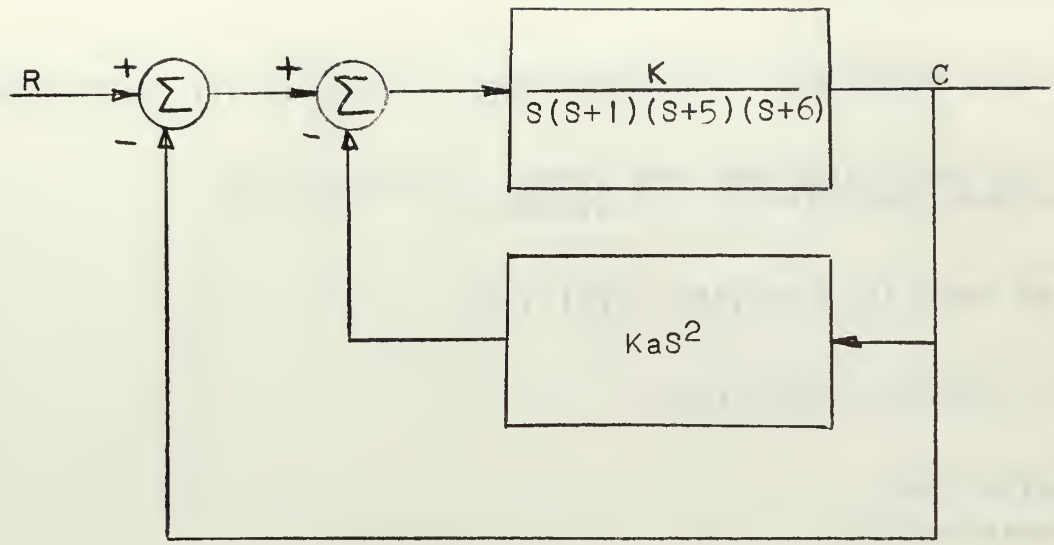


Figure 4.3

Redesign of Example 4.2.

Root Specifications: Zeta = 0.5, Natural Frequency = 12.0

No Velocity Constant and Bandwidth Specified

```
INPUT DATA
**********

POLES ARE ZERO,    0.10000D 01,   0.50000D 01,   0.60000D 01

ROOT SPECIFICATIONS ARE ZETA=    0.50000D 00,
NATURAL FREQUENCY=    0.12000D 02

 NO VELOCITY CONSTANT SPECIFIED

NO BANDWIDTH SPECIFIED

OUTPUT DATA
***********

FORWARD GAIN

    0.36000E 03

ACCELERATION GAIN

    0.29306E 00

COEFFICIENTS OF CHARACTERISTIC EQUATION
(ASCENDING ORDER)

    0.36000E 03    0.30000E 02    0.14650E 03    0.12000E 02
    0.10000E 01

 THE SYSTEM ROOTS ARE

 REAL PART

   -0.60000E 01   -0.60000E 01   -0.37068E-07   -0.37068E-07

 IMAGINARY PART

   -0.10392E 02    0.10392E 02   -0.15811E 01    0.15811E 01
```

Examole 4.4

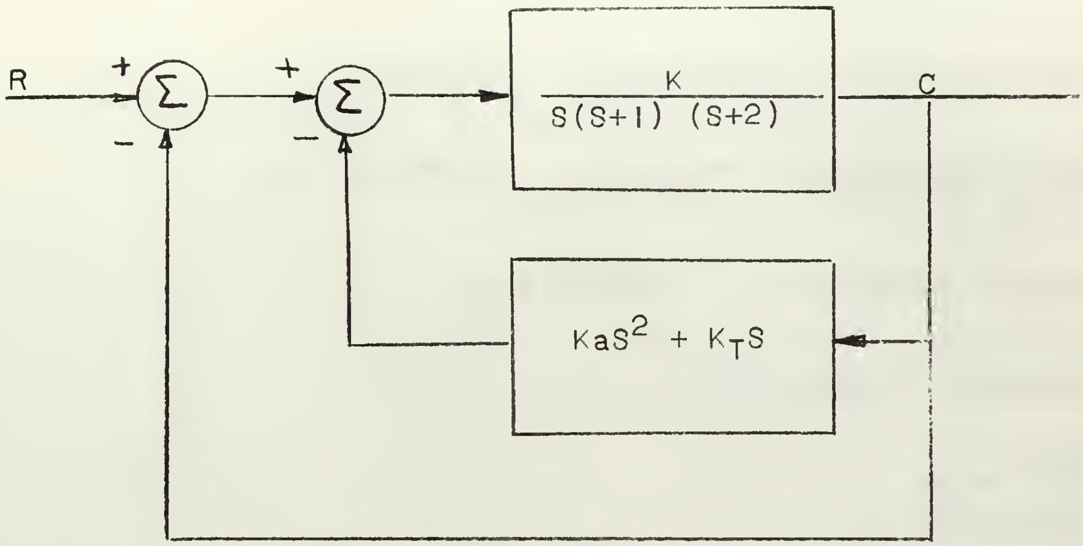


Figure 4.4

Design a third order system with acceleration and tachometer feedback for the open loop system in Figure 4.4.

Root Specification: Zeta = 0.7, Natural Frequency = 10.0

Velocity Constant = 6.0

No Bandwidth Specification

INPUT DATA
****

POLES ARE ZERO,   0.10000D 01,   0.20000D 01

ROOT SPECIFICATIONS ARE ZETA=   0.70000D 00,
NATURAL FREQUENCY=   0.10000D 02

VELOCITY CONSTANT IS   0.60000D 01

NO BANDWIDTH SPECIFIED

OUTPUT DATA

****

FORWARD GAIN

0.37500E 04

TACHOMETER GAIN

0.16613E 00

ACCELERATION GAIN

0.12933E-01

COMPUTED VELOCITY CONSTANT IS =   0.60000E 01

COEFFICIENTS OF CHARACTERISTIC EQUATION,
(ASCENDING ORDER)

0.37500E 04   0.62500E 03   0.51500E 02   0.10000E 01

THE SYSTEM ROCTS ARE

REAL PART

-0.70000E 01   -0.70000E 01   -0.37500E 02

IMAGINARY PART

0.71414E 01   -0.71414E 01   0.0

Example 4.5

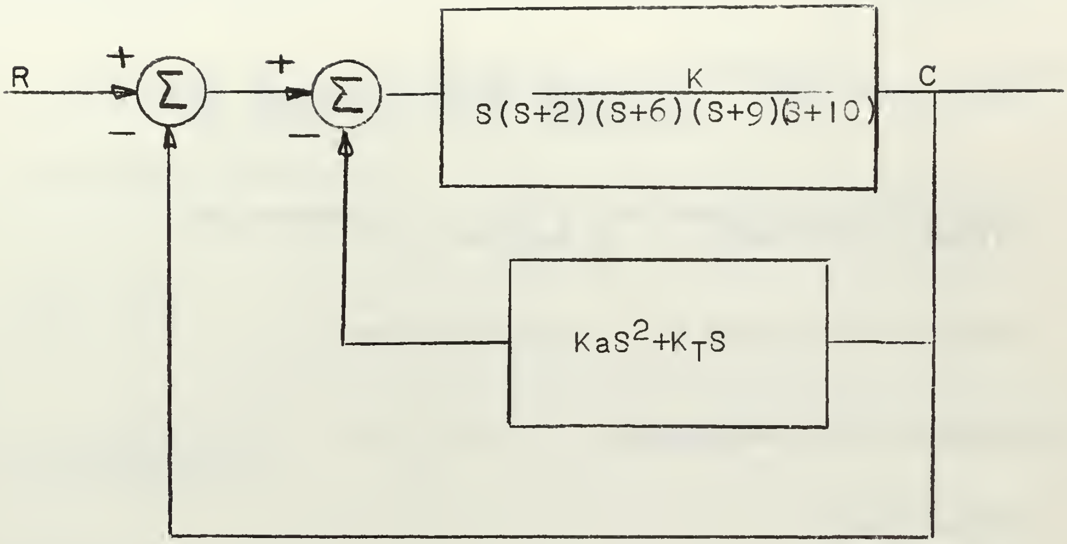


Figure 4.5

Design a fifth order system with acceleration and tachometer feedback for the open loop system Figure 4.5.

Root Specification: Zeta = 0.5, Natural Frequency = 4.0

Velocity Constant = 0.5

No Bandwidth Specification

```
INPUT DATA
**********

POLES ARE ZERO,    0.20000D 01,    0.60000D 01
                   0.90000D 01,    0.10000D 02

ROOT SPECIFICATIONS ARE ZETA=    0.50000D 00,
NATURAL FREQUENCY=    0.40000D 01

VELOCITY CONSTANT IS    0.50000D 00

NO BANDWIDTH SPECIFIED

OUTPUT DATA
***********

FORWARD GAIN

    0.13349E 04

TACHOMETER GAIN

    0.11909E 01

ACCELERATION GAIN

    0.65497E-01

COMPUTED VELOCITY CONSTANT IS =    0.50000E 00
```

```
COEFFICIENTS OF CHARACTERISTIC EQUATION
(ASCENDING ORDER)

    0.13349E 04     0.26697E 04     0.10354E 04     0.25400E 03
    0.27000E 02     0.10000E 01

 THE SYSTEM ROCTS ARE



 REAL PART


  -0.63277E 0C    -0.11184E 02    -0.11184E 02    -0.20000E 01
  -0.20000E 01


 IMAGINARY PART


    0.0           -0.26026E 01     0.26026E 01    -0.34641E 01
    0.34641E 01
```

Example 4.6

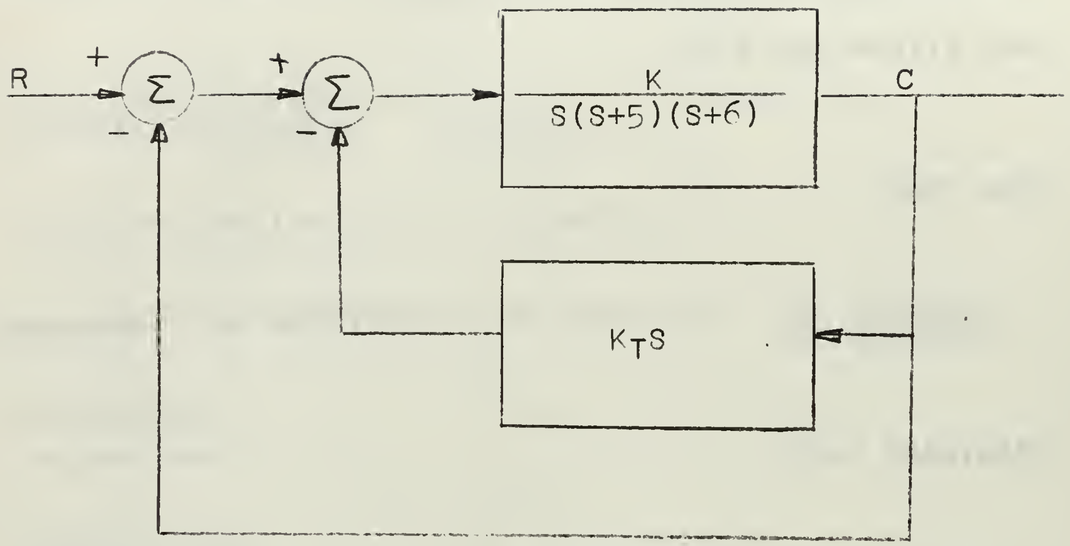


Figure 4.6

Design a tachometer feedback system for the open loop system in Figure 4.6.

No Root Specification

Velocity Constant = 0.91

Bandwidth Frequency = 4.0

Systems roots are for number two gain values since system gain one is negative.

```
INPUT DATA
**********


POLES ARE ZERO,    0.50000E 01,    0.60000E 01

 NO ROOT SPECIFICATION

 VELOCITY CONSTANT IS=    0.91000E 00

BANDWIDTH FREQUENCY IS=    0.40000E 01

OUTPUT DATA
***********


SYSTEM GAIN ONE=   -0.31554E 02
TACHOMETER GAIN ONE=    0.20496E 01


SYSTEM GAIN TWO=    0.41920E 02
TACHOMETER GAIN TWO=    0.38325E 00

COEFFICIENTS OF CHARACTERISTIC EQUATION
(ASCENDING ORDER)

    0.41920E 02    0.46066E 02    0.11000E 02    0.10000E 01

THE SYSTEM ROOTS ARE


REAL PART

   -0.12317E 01   -0.48841E 01   -0.48841E 01


IMAGINARY PART

    0.0           -0.31905E 01    0.31905E 01
```

Example 4.7

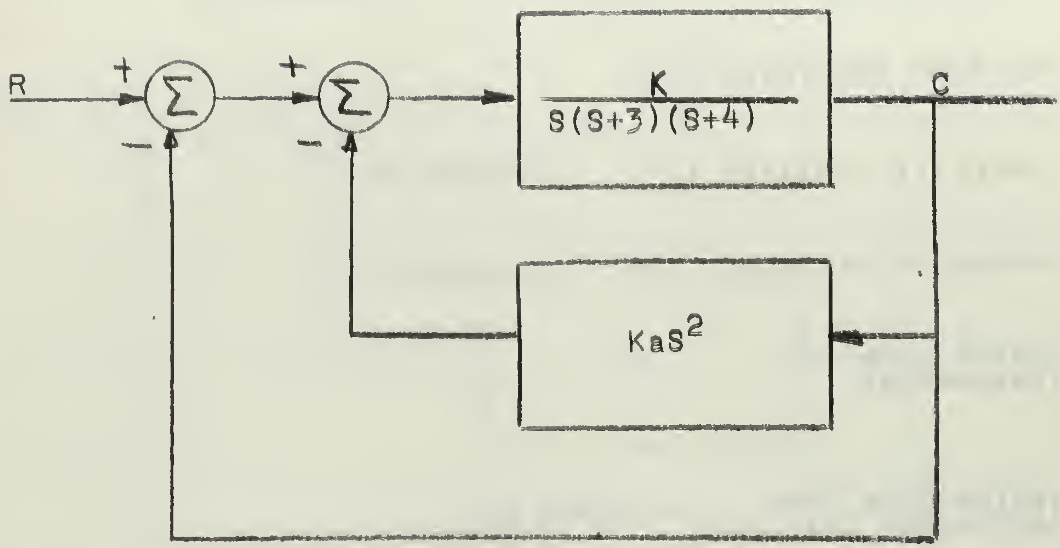


Figure 4.7

Design an accelerometer feedback system for the open loop system Figure 4.7.

No Root Specification

Velocity Constant = 5.0

Bandwidth Frequency = 3.0

System roots are for gain values number one since Ka2 is negative.

INPUT DATA
****

POLES ARE ZERO, 0.30000E 01, 0.40000E 01

NO ROOT SPECIFICATION

VELOCITY CONSTANT IS= 0.50000E 01

BANDWIDTH FREQUENCY IS= 0.30000E 01

OUTPUT DATA
****

SYSTEM GAIN ONE= 0.60000E 02
ACCELERATION GAIN ONE= 0.49388E-01

SYSTEM GAIN TWO= 0.60000E 02
ACCELERATION GAIN TWO= -0.50494E 00

COEFFICIENTS OF CHARACTERISTIC EQUATION
(ASCENDING ORDER)

0.60000E 02 0.12000E 02 0.99633E 01 0.10000E 01

THE SYSTEM ROOTS ARE

REAL PART

-0.29863E 00 -0.29863E 00 -0.93660E 01

IMAGINARY PART

0.25134E 01 -0.25134E 01 0.0

Example 4.8

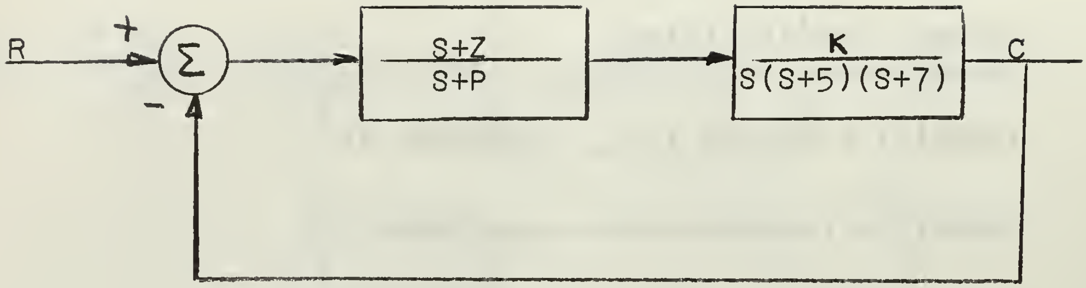


Figure 4.8

Compensate the open loop system in Figure 4.8 using a single section cascade compensator.

Root Specification: Zeta = 0.5, Natural Frequency = 6.0

Velocity Constant = 4.15

No Bandwidth Specification

```
INPUT DATA
**********


POLES ARE ZERO,    0.50000D 00,    0.70000D 01


ROOT SPECIFICATIONS ARE ZETA=    0.50000D 00,
NATURAL FREQUENCY=    0.60000D 01


VELOCITY CONSTANT IS    0.41500D 01


NO BANDWIDTH SPECIFIED


OUTPUT DATA
***********


SYSTEM GAIN


    0.13756E 04


COMPENSATOR POLE


    0.37847E 02


COMPENSATOR ZERO


    0.39961E 0C
```

VELOCITY CONSTANT(COMPUTED)

0.41500E 01

COEFFICIENTS OF CHARACTERISTIC EQUATION
(ASCENDING ORDER)

0.54972E 03	0.15081E 04	0.28735E 03
0.45347E 02	0.10000E 01	

THE SYSTEM ROCTS ARE

REAL PART

-0.39200E CC	-0.30000E 01	-0.30000E 01	-0.38955E 02

IMAGINARY PART

0.0	-0.51962E 01	0.51962E 01	0.0

INPUT DATA
****

POLES ARE ZERO,   0.10000D 01,   0.15000D 02

ROOT SPECIFICATIONS ARE ZETA=   0.66500D 00,
NATURAL FREQUENCY=   0.59600D 01

VELOCITY CONSTANT IS=   0.28000D 02

NO BANDWIDTH SPECIFIED

ROOT SPECIFICATIONS ARE ZETA=   0.50600D 00,
NATURAL FREQUENCY=   0.14660D 02

OUTPUT DATA
****

FORWARD GAIN

0.34637E 04

FILTER ZEROS ARE

REAL PART

-0.51508E 01   -0.91788E 01

IMAGINARY PART

0.0   0.0

Example 4.9

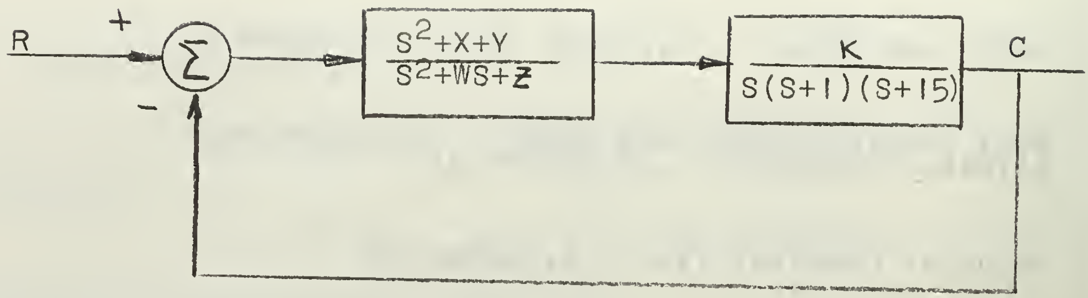


Figure 4.9

Design a double section compensator for the open loop system Figure 4.9.

Root Specifications: Zeta 1 = 0.665, Natural Frequency 1 = 5.96; Zeta 2 = 0.506, Natural Frequency 2 = 14.66

Velocity Constant = 28.0

No Bandwidth Specification

FILTER POLES ARE

REAL PART

```
-0.14107E 02   -0.14107E 02
```

IMAGINARY PART

```
-0.13817E 02    0.13817E 02
```

COEFFICIENTS OF CHARACTERISTIC EQUATION
(ASCENDING ORDER)

```
 0.16376E 06    0.55482E 05    0.10125E 05
 0.85632E 03    0.44213E 02    0.10000E 01
```

THE SYSTEM ROCTS ARE

REAL PART

```
-0.39634E 01   -0.39634E 01   -0.21451E 02   -0.74180E 01
-0.74180E 01
```

IMAGINARY PART

```
-0.44512E 01    0.44512E 01    0.0            -0.12645E 02
 0.12645E 02
```

Example 4.10

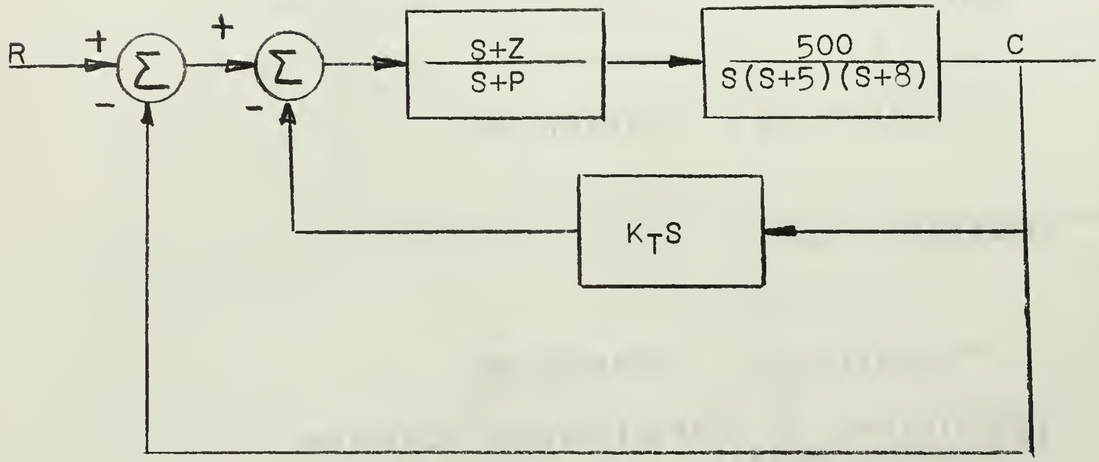


Figure 4.10

Design a single section cascade compensator with tachometer feedback for open loop system Figure 4.10.

Root Specification: Zeta = 0.5, Natural Frequency = 7.0

Velocity Constant = 0.5

No Bandwidth Specified

These specifications can not be satisfied. Choice one gives negative feedback gain and roots indicated. Choice two gives undesirable right half plane compensator.

INPUT DATA
****

SYSTEM POLES ARE ZERO,,   0.50000E 01,   0.80000E 01

ROOT SPECIFICATIONS ARE ZETA=   0.50000E 00,
NATURAL FREQUENCY=   0.70000E 01

VELOCITY CONSTANT IS =   0.50000E 00

NO BANDWIDTH SPECIFIED

OUTPUT DATA
****

CHOICE ONE

TACHOMETER GAIN=   -0.73364E-01
COMPENSATOR POLE=   0.15640E 02
COMPENSATOR ZERO=   0.60346E 00

CHOICE TWO

TACHOMETER GAIN=   0.60888E 00
COMPENSATOR POLE=   -0.46819E 02
COMPENSATOR ZERO=   -0.26924E 01

COEFFICIENTS OF CHARACTERISTIC EQUATION
(ASCENDING ORDER)

0.30173E 03   0.11035E 04   0.20664E 03   0.28640E 02
0.10000E 01

THE SYSTEM ROOTS ARE

REAL PART

-0.28840E 00   -0.35000E 01   -0.35000E 01   -0.21352E 02

IMAGINARY PART

0.0   -0.60622E 01   0.60622E 01   0.0

Example 4.11

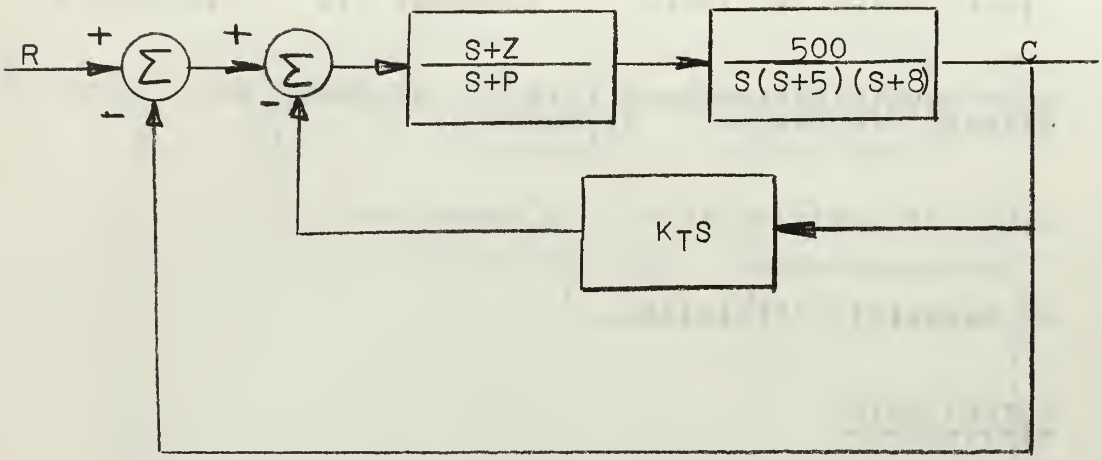


Figure 4.11

Redesign Example 4.10 for single section cascade compensator with tachometer feedback for open loop system Figure 4.11.

Root Specifications: Zeta = 0.5, Natural Frequency = 10.0

Velocity Constant = 0.5

No Bandwidth Specified

Data on page 61 shows results for Wn = 10.0 and system roots for parameter values choice one. If Wn is changed to Wn = 14.0 data on page 62 shows both choices of parameter values are satisfactory.

INPUT DATA
****

SYSTEM POLES ARE ZERO,, 0.50000E 01, 0.80000E 01

ROOT SPECIFICATIONS ARE ZETA= 0.50000E 00,
NATURAL FREQUENCY= 0.10000E 02

VELOCITY CONSTANT IS = 0.50000E 00

NO BANDWIDTH SPECIFIED

OUTPUT DATA
****

CHOICE ONE

TACHOMETER GAIN= 0.16103E 00
COMPENSATOR POLE= 0.34088E 01
COMPENSATOR ZERO= 0.14829E 00

CHOICE TWO

TACHOMETER GAIN= 0.12470E 01
COMPENSATOR POLE= -0.21609E 03
COMPENSATOR ZERO= -0.22957E 02

COEFFICIENTS OF CHARACTERISTIC EQUATION
(ASCENDING ORDER)

0.74145E 02 0.64829E 03 0.16483E 03 0.16409E 02
0.10000E 01

THE SYSTEM ROOTS ARE

REAL PART

-0.11786E 00	-0.62909E 01	-0.50000E 01	-0.50000E 01

IMAGINARY PART

0.0	0.0	-0.86603E 01	0.86603E 01

INPUT DATA
****

SYSTEM POLES ARE ZERO,, 0.50000E 01, 0.80000E 01

ROOT SPECIFICATIONS ARE ZETA= 0.50000E 00,
NATURAL FREQUENCY= 0.14000E 02

VELOCITY CONSTANT IS = 0.50000E 00

NO BANDWIDTH SPECIFIED

OUTPUT DATA
****

CHOICE ONE

TACHOMETER GAIN= 0.29434E 00
COMPENSATOR POLE= 0.46165E 01
COMPENSATOR ZERO= 0.21652E 00

CHOICE TWO

TACHOMETER GAIN= 0.16032E 01
COMPENSATOR POLE= 0.43558E 03
COMPENSATOR ZERO= 0.87812E 02

COEFFICIENTS OF CHARACTERISTIC EQUATION
(ASCENDING ORDER)

0.10826E 03 0.71652E 03 0.24718E 03 0.17616E 02
0.10000E 01

THE SYSTEM ROOTS ARE

REAL PART

-0.15980E 00 -0.34564E 01 -0.70001E 01 -0.70001E 01

IMAGINARY PART

0.0 0.0 -0.12124E 02 0.12124E 02

Example 4.12

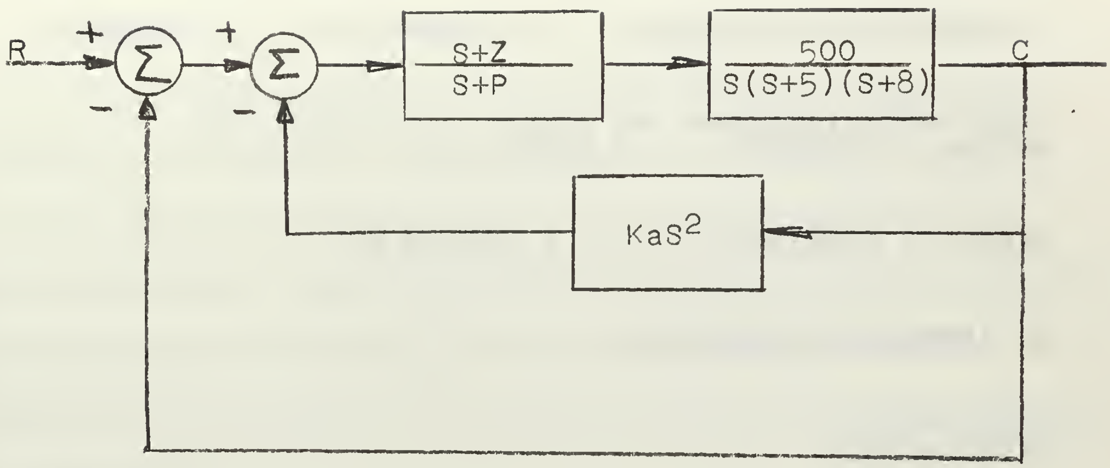


Figure 4.12

Design single section cascade compensator with acceleration feedback for open loop sy~tem Figure 4.12.

Root Specifications: Zeta = 0.5, Natural Frequency = 5.0

Velocity Constant = 0.5

No Bandwidth Specified

System roots are for choice two parameters

```
INPUT DATA
**********


SYSTEM POLES ARE ZERO,,    0.50000E 01,    0.80000E 01


ROOT SPECIFICATIONS ARE ZETA=    0.50000E 00,
NATURAL FREQUENCY=    0.50000E 01


VELOCITY CONSTANT IS =    0.50000E 00


NO BANDWIDTH SPECIFIED


OUTPUT DATA
***********

CHOICE ONE

ACCELERATION GAIN=    -0.26303E 00
COMPENSATOR POLE=     0.20500E-44
COMPENSATOR ZERO=    -0.13047E 02


CHOICE TWO

ACCELERATION GAIN=    0.37030E-01
COMPENSATOR POLE=     0.14807E 02
COMPENSATOR ZERO=     0.59227E 00


COEFFICIENTS OF CHARACTERISTIC EQUATION
(ASCENDING ORDER)


    0.29614E 03    0.10923E 04    0.24345E 03    0.46322E 02
    0.10000E 01


THE SYSTEM ROOTS ARE
REAL PART


   -0.28868E 00   -0.25000E 01   -0.25000E 01   -0.41033E 02

IMAGINARY PART


    0.0           -0.43301E 01    0.43301E 01    0.0
```

V. General Program

The problems solved in this thesis were done on an individual basis with no attempt made to write a general program. In retrospect it is seen that a general program could be written provided the type of specifications to be satisfied are selected, the maximum system order including any compensation effects is stated, the desired output information and format is known, and the system configuration is considered.

Our prime specification was a complex root specification which is dependent on the form of the characteristic equation coefficients. In all the cases considered the right hand side of the matrix equation had the same form. The form is a function of system type and order, and has two general forms

$$\sum_{K=0}^{n} -a_K \omega_n^K \phi_{K-1}(S) \tag{5-1}$$

from equation (2-3) and

$$\sum_{K=0}^{n} -a_K \omega_n^K \phi_K(S) \tag{5-2}$$

from equation (2-4) where

$$a_n = 1.0$$

$$a_{n-1} = \text{Sum of All Poles}$$

$$a_{n-i} = \text{Sum of Product of } i \text{ Poles} \quad , \tag{5-3}$$

The left hand side coefficient matrix also reduces to a convenient form

$$\begin{bmatrix} \phi_{-1}(\rho) & \omega_n \phi_0(\rho) \cdots & \omega_n^j \phi_{j-1}(\rho) \\ \phi_0(\rho) & \omega_n \phi_1(\rho) \cdots & \omega_n^j \phi_j(\rho) \end{bmatrix} \begin{bmatrix} R_1 \\ \vdots \\ R_j \end{bmatrix} \qquad (5\text{-}4)$$

where R is an unknown appearing in the j^{th} coefficient of the characteristic equation. If a particular coefficient of the characteristic equation has no unknowns then the appropriate place in the coefficient matrix above is replaced by a zero. As noticed for the cascade compensation system terms similar to the right hand side are generated in the left hand coefficient matrix multiplying the unknown filter pole or polynomial coefficient except the coefficients are

$$\sum_{K-n}^{n} a_K \omega_n^K \phi_K(\rho) \qquad (5\text{-}5)$$

and

$$\sum_{K-n}^{n} a_K \omega_n^K \phi_{K-1}(\rho) \qquad (5\text{-}6)$$

where

$K = n +$ ORDER OF FILTER

$a_n = 1.0$

$a_{n-1} = \sum_{i=0}^{K-n} P_i$

$a_{n-i} =$ SUM OF THE PRODUCT OF i OF THE FILTER POLES $i = 1, \cdots, K-n$

The root equations can be generalized by applying them to a characteristic equation with coefficients

$$A_K = a_K \alpha + b_K \beta + c_K \qquad (5\text{-}7)$$

and then the result can be reduced to a given situation by reading in the proper value for the coefficients α_K, B_K and C_K.

The velocity constant also has certain properties that make it conducive to programming. The basic form is

$$K_v = \frac{K \prod^{m} z_i}{\prod^{n} P_i} \tag{5-8}$$

where n is the number of system poles and m is the number of system zeros. The form above applies for systems with unity feedback and acceleration feedback. If tachometer feedback is used the velocity constant becomes

$$K_v = \frac{K \prod^{m} z_i}{\prod^{n} P_i + KK_t \prod^{m} z_i} \tag{5-9}$$

The remaining situations are for cascaded systems and the form is now

$$K_v = \frac{K \prod^{m} z_i \prod^{K} z_j}{\prod^{n} P_i \prod^{K} P_j} \tag{5-10}$$

where k is the number of cascade filter sections. The cascade acceleration combination is as above but the tachometer cascade combination becomes

$$K_v = \frac{K \prod^{m} z_i \prod^{K} z_j}{\prod^{n} P_i \prod^{K} P_j + K \prod^{m} z_i \prod^{K} z_j} \tag{5-11}$$

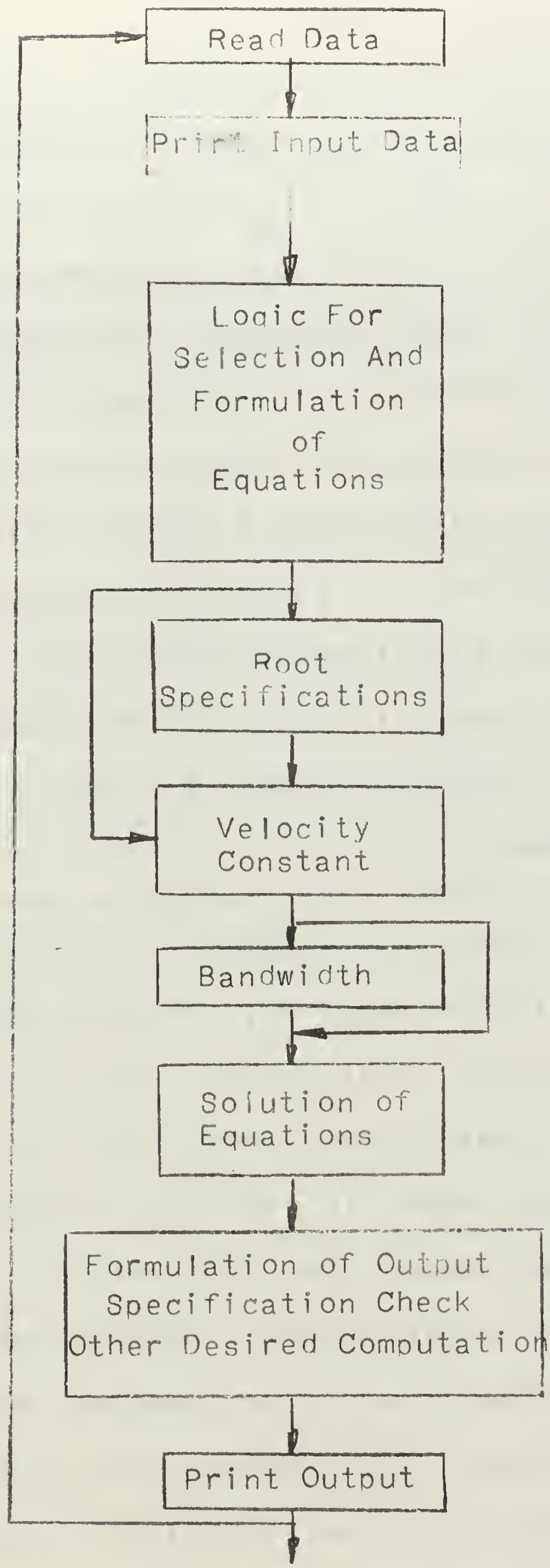
In all the above situations if any pole or zero is not in the system it is replaced by a one where as removal of tachometer feedback implies $K_t = 0$.

The final specification presents a rather tedious problem as the general form shows

$$\frac{1}{\sqrt{2}} = \left| \frac{K\left(s^{m} + b_{m-1} s^{m-1} + \cdots + b_0\right)}{s\left(a_{n-1} s^{n-2} + a_{n-3} s^{n-4} + \cdots + 1\right) + \left(a_n s^{n} + a_{n-2} s^{n-3} + \cdots + a_0\right)} \right|_{s=j\omega_b} \quad (5-12)$$

where the a_k's are a function of the unknowns. Since the terms are squared it is recommended that bandwidth be set up for the most difficult case expected to be encountered.

Once the specifications have been preset in the program they can be implemented by reading in appropriate constants, then using logic statements the simultaneous matrix equations can be formed. Again as in the bandwidth case the nonlinear cases using the elimination method for solution should only be set up for the maximum system configuation and order expected to be encountered. This is because the computer can process numbers but is unable to process unknowns specified by alphabetic characters. Designers may find that for cases of this nature it may be beneficial to use an appropriate iteration routine mentioned in the next section. A rough general flow chart of major operations is shown in Figure 5-1.



General Program Flowgraph
Figure 5-1

VI. Conclusions and Recommendations

The purpose of this paper was to investigate the feasibility of using a computer for designing with algebraic methods. It has been shown that this can be accomplished, in most cases, by the linearizing transformation of variables or manipulation of system analytic form. This was not true for the feedback-cascade compensation cases or when bandwidth specification was to be satisfied but these problems could be solved by application of a straight elimination process.

The important advantage of this method is it only restrains the system sufficiently to solve for the unknowns. Other design attempts tended to specify all root locations in implementing a solution.

An area of possible future study is converting other system specifications into algebraic equations and analyzing techniques of solution required. We have dealt with only three specifications in this paper. Can we effectively use peak overshoot, steady state error, settling time, resonance frequency, or rise time? In analyzing these specifications nonlinearities may appear that defy solution by elimination, therefore, it may require using iteration methods for solution like Newton's method or steepest decent, or a combination of these. The results obtained by these methods are dependent heavily on the initial value guess and to a lesser degree the quality of convergence to the final value.

An interesting point for further investigation would be the possibility of deriving another definition for bandwidth based on a criteria other than magnitude. If bandwidth can be specified as a complex number then the non-linearities are not compounded by squaring and the specification will reduce to a two equations similar to the root equations.

BIBLIOGRAPHY

1. Siljak, D. D., Analysis and Synthesis of Feedback Control Systems in the Parameter Plane, I-Linear Continuous Systems. IEEE Transactions on Application and Industry, vol. 83, No. 75, November, 1964, pp. 449-458.

2. Thaler, G. J., Siljak, D. D. and Dorf, R. C. Algebraic Methods for Dynamic Systems. Interim Technical Report Prepared for NASA Under Contract NAS2-2609, 1966.

3. Nutting, R. M., Parameter Plane Techniques for Feedback Control Systems. M.S. in EE Thesis, Naval Postgraduate School, 1965.

4. Siljak, D. D., Generalization of the Parameter Plane Method. IEEE Transactions on Automatic Control, vol. II, No. I, January, 1966, pp. 65-70.

5. Thaler, G. J., Elliot, D. W. and Heseltine, J. C. W. Feedback Compensation Using Derivative Signals, II-Mitrovic's Method. AIEE Transactions on Application and Industry, September, 1963.

6. Zaguskin, V. L., Handbook of Numerical Methods for the Solution of Algebraic and Transcendental Equations, Pergamon Press, 1961.

APPENDIX I

$\phi_k(\ \ )$ FUNCTIONS

	ϕ_{-1}	ϕ_0	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7
0.00	1	0	-1	0.0	1.00	0.000	-1.0000	0.00000	1.000000
0.05	1	0	-1	0.1	0.99	-0.199	-0.9701	0.29601	0.940499
0.10	1	0	-1	0.2	0.96	-0.392	-0.8816	0.56832	0.767936
0.15	1	0	-1	0.3	0.91	-0.573	-0.7381	0.79443	0.499771
0.20	1	0	-1	0.4	0.84	-0.736	-0.5456	0.95424	0.163904
0.25	1	0	-1	0.5	0.75	-0.875	-0.3125	1.03125	-0.203125
0.30	1	0	-1	0.6	0.64	-0.984	-0.0496	1.01376	-0.558656
0.35	1	0	-1	0.7	0.51	-1.057	0.2299	0.89607	-0.857149
0.40	1	0	-1	0.8	0.36	-1.088	0.5104	0.67960	-1.054144
0.45	1	0	-1	0.9	0.19	-1.071	0.7739	0.37449	-1.110941
0.50	1	0	-1	1.0	0.00	-1.000	1.0000	0.00000	-1.000000
0.55	1	0	-1	1.1	-0.21	-0.869	1.1659	-0.41349	-0.711061
0.60	1	0	-1	1.2	-0.44	-0.672	1.2464	-0.82368	-0.257984
0.65	1	0	-1	1.3	-0.69	-0.403	1.2139	-1.17507	0.313691
0.70	1	0	-1	1.4	-0.96	-0.056	1.0384	-1.39776	0.918464
0.75	1	0	-1	1.5	-1.25	0.375	0.6875	-1.40625	1.421875
0.80	1	0	-1	1.6	-1.56	0.896	0.1264	-1.09824	1.630784
0.85	1	0	-1	1.7	-1.89	1.513	-0.6821	-0.35343	1.282931
0.90	1	0	-1	1.8	-2.24	2.232	-1.7776	0.96768	0.035776
0.95	1	0	-1	1.9	-2.61	3.059	-3.2021	3.02499	-2.545381
1.00	1	0	-1	2.0	-3.00	4.000	-5.0000	6.00000	-7.000000

APPENDIX II

LISTING OF PROGRAMS

```
C       THIS PROGRAM USES ALGEBRAIC METHODS TO SOLVE FEEDBACK COMPENSATION
C       WITH ROOT SPECIFICATOONS,VELOCITY ERROR AND BANDWIDTH, THE ROOTS
C       ARE SPECIFIED BY A ZETA AND AND OMEGA, THE VELOCITY ERROR BY THE
C       ERROR CONSTANT KV AND BANDWIDTH BY THE 3DB FREQUENCY WB. THIS
C       IS NOT A GENERAL PROGRAM, IT IS IN FORTRAN FOUR USING A IBM 360
        REAL*8 A,B,PHIO,PHI1,PHI2,PHI3,PHI4,PHI5,P,D,E,ZETA,AMEGA,F,VELCON
       &,AMEGB
        DIMENSION B(3),A(3,3),P(4),D(2),E(2,2),R(3,3),C(5),W(5),ROOTR(5),R
       &OOTI(5),F(4)
C       THE POLES HAVE TO BE PUT IN AS FOLLOWS, LHP POLES WITH PLUS SIGN,
C       RHP POLES WITH A MINUS SIGN, NOTE MUST BE TYPE ONE SYSTEM
    999 READ(5,1) P(1),P(2)
        READ(5,1) P(3),P(4)
      1 FORMAT(2D15.5)
        READ(5,1) ZETA,AMEGA
        READ(5,1) VELCON,AMEGB
C       IF TACHOMETER FEEDBACK T=1,ACCELERATION FEEDBACK T=2,TACH AND ACCE
C       FEEDBACK T=3
        READ(5,2) T
      2 FORMAT(E15.5)
        PHIO=0.0
        PHI1=-1.0
        PHI2=2.0*ZETA
        PHI3=1.0-4.0*ZETA**2
        PHI4=-4.0*ZETA+8.0*ZETA**3
        PHI5=-1.0+12.0*ZETA**2-16.0*ZETA**4
        IF(T.LT.2.0) GO TO 100
        IF(T.LT.3.0) GO TO 200
        IF(T.LT.4.0) GO TO 300
    100 WRITE(6,110)
    110 FORMAT(1H1,//,20X,'INPUT DATA',/)
        WRITE(6,502)
    502 FORMAT(20X,'**********',/)
        WRITE(6,111) P(1),P(2)
    111 FORMAT(//,20X,'POLES ARE ZERO,',D15.5,',',D15.5,/)
        WRITE(6,112) ZETA,AMEGA
    112 FORMAT(//,20X,'ROOT SPECIFICATIONS ARE ZETA=',D15.5,',',/,20X,'NAT
       &URAL FREQUENCY=',D15.5,/)
        WRITE(6,113)
    113 FORMAT(//,20X,' NO VELOCITY CONSTANT SPECIFIED',/)
        WRITE(6,114)
    114 FORMAT(//,20X,'NO BANDWIDTH SPECIFIED',/)
        D(1)=-P(1)*P(2)*AMEGA*PHIO-(P(1)+P(2))*AMEGA**2*PHI1-PHI2*AMEGA**3
        D(2)=-P(1)*P(2)*AMEGA*PHI1-(P(1)+P(2))*AMEGA**2*PHI2-PHI3*AMEGA**3
        E(1,1)=-PHI1
```

```
      E(1,2)=AMEGA*PHIO
      E(2,1)=PHIO
      E(2,2)=AMEGA*PHI1
      CALL DSIMQ(E,D,2,KS)
      GAIN=D(1)
      TGAIN=D(2)/D(1)
      WRITE(6,115)
  115 FORMAT(//,20X,'OUTPUT DATA',/)
      WRITE(6,501)
  501 FORMAT(20X,'************',/)
      WRITE(6,101)
  101 FORMAT(//,20X,12HFORWARD GAIN,/)
      WRITE(6,102) GAIN
  102 FORMAT(/,20X,E15.5,/)
      WRITE(6,103)
  103 FORMAT(//,20X,15HTACHOMETER GAIN,/)
      WRITE(6,102) TGAIN
      DO 104 I=1,4
  104 W(I)=0.0
      PO1=P(1)
      PO2=P(2)
      C(1)=GAIN
      C(2)=PO1*PO2+GAIN*TGAIN
      C(3)=PO1+PO2
      C(4)=1.0
      WRITE(6,105)
  105 FORMAT(//,20X,'COEFFICIENTS OF CHARACTERISTIC EQUATION',/,20X,'(AS
     &CENDING ORDER)')
      WRITE(6,199) (C(I),I=1,4)
  199 FORMAT(//,20X,4E15.5,/)
      DO 106 J=1,3
      ROOTR(J)=0.0
  106 ROOTI(J)=0.0
      CALL POLRT(C,W,3,ROOTR,ROOTI,IER)
      WRITE(6,107)
  107 FORMAT(//,20X,21H THE SYSTEM ROOTS ARE,/)
      WRITE(6,108)
  108 FORMAT(////,20X,10H REAL PART,//)
      WRITE(6,116) (ROOTR(I),I=1,3)
  116 FORMAT(//,20X,4E15.5,/,20X,4E15.5,/)
      WRITE(6,117)
  117 FORMAT(////,20X,15H IMAGINARY PART,//)
      WRITE(6,116) (ROOTI(I),I=1,3)
      WRITE(6,600)
  600 FORMAT(/,1H1,/)
```

```
      GO TO 999
  200 WRITE(6,110)
      WRITE(6,502)
      WRITE(6,221) P(1),P(2),P(3)
  221 FORMAT(//,20X,'POLES ARE ZERO,',D15.5,',',D14.5,',',D14.5,/)
      WRITE(6,112) ZETA,AMEGA
      WRITE(6,113)
      WRITE(6,114)
      D(1)=-P(1)*P(2)*P(3)*AMEGA*PHI0-(P(1)*P(2)+P(2)*P(3)+P(1)*P(3))*AM
     &EGA**2*PHI1-(P(1)+P(2)+P(3))*AMEGA**3*PHI2-AMEGA**4*PHI3
      D(2)=-P(1)*P(2)*P(3)*AMEGA*PHI1-(P(1)*P(2)+P(2)*P(3)+P(1)*P(3))*AM
     &EGA**2*PHI2-(P(1)+P(2)+P(3))*AMEGA**3*PHI3-AMEGA**4*PHI4
      E(1,1)=-PHI1
      E(1,2)=PHI1*AMEGA**2
      E(2,1)=PHI0
      E(2,2)=PHI2*AMEGA**2
      CALL DSIMQ(E,D,2,KS)
      GAIN=D(1)
      AGAIN=D(2)/D(1)
      WRITE(6,115)
      WRITE(6,501)
      WRITE(6,201)
  201 FORMAT(//,20X,12HFORWARD GAIN,/)
      WRITE(6,202) GAIN
  202 FORMAT(/,20X,E15.5,/)
      WRITE(6,203)
  203 FORMAT(//,20X,17HACCELERATION GAIN,/)
      WRITE(6,202) AGAIN
      DO 222 I=1,5
  222 W(I)=0.0
      PO1=P(1)
      PO2=P(2)
      PO3=P(3)
      C(1)=GAIN
      C(2)=PO1*PO2*PO3
      C(3)=PO1*PO2+PO1*PO3+PO2*PO3+GAIN*AGAIN
      C(4)=PO1+PO2+PO3
      C(5)=1.0
      WRITE(6,105)
      WRITE(6,223) (C(I),I=1,5)
  223 FORMAT(//,20X,4E15.5,/,20X,4E15.5,/)
      DO 224 J=1,4
      ROOTR(J)=0.0
  224 ROOTI(J)=0.0
      CALL POLRT(C,W,4,ROOTR,ROOTI,IER)
```

```
      WRITE(6,107)
      WRITE(6,108)
      WRITE(6,116)  (ROOTR(I),I=1,4)
      WRITE(6,117)
      WRITE(6,116)  (ROOTI(I),I=1,4)
      WRITE(6,600)
      GO TO 999
  300 WRITE(6,110)
      WRITE(6,502)
      WRITE(6,321) P(1),P(2),P(3),P(4)
  321 FORMAT(//,20X,'POLES ARE ZERO,',D14.5,',',D14.5,/,35X,D14.5,',',D1
     &4.5,/)
      WRITE(6,112) ZETA ,AMEGA
      WRITE(6,322) VELCON
  322 FORMAT(//,20X,'VELOCITY CONSTANT IS',D15.5,/)
      WRITE(6,114)
      F(1)=P(1)+P(2)+P(3)+P(4)
      F(2)=P(1)*P(2)+P(2)*P(3)+P(1)*P(3)+P(1)*P(4)+P(2)*P(4)+P(3)*P(4)
      F(3)=P(1)*P(2)*P(3)+P(1)*P(2)*P(4)+P(1)*P(3)*P(4)+P(2)*P(3)*P(4)
      F(4)=P(1)*P(2)*P(3)*P(4)
      B(1)=-F(4)*VELCON
      B(3)=-F(4)*AMEGA*PHIO-F(3)*AMEGA**2*PHI1-F(2)*AMEGA**3*PHI2-F(1)*A
     &MEGA**4*PHI3-AMEGA**5*PHI4
      B(2)=-F(4)*AMEGA*PHI1-F(3)*AMEGA**2*PHI2-F(2)*AMEGA**3*PHI3-F(1)*A
     &MEGA**4*PHI4-AMEGA**5*PHI5
      A(1,1)=-1.0
      A(1,2)=VELCON
      A(1,3)=0.0
      A(2,1)=PHIO
      A(2,2)=PHI1*AMEGA
      A(2,3)=PHI2*AMEGA**2
      A(3,1)=-PHI1
      A(3,2)=PHIO*AMEGA
      A(3,3)=PHI1*AMEGA**2
      CALL DSIMQ(A,B,3,KS)
      GAIN=B(1)
      TGAIN=B(2)/B(1)
      AGAIN=B(3)/B(1)
      WRITE(6,115)
      WRITE(6,501)
      WRITE(6,301)
  301 FORMAT(//,20X,12HFORWARD GAIN,/)
      WRITE(6,302) GAIN
  302 FORMAT(/,20X,E15.5,/)
      WRITE(6,303)
```

```
  303 FORMAT(//,20X,15HTACHOMETER GAIN,/)
      WRITE(6,302) TGAIN
      WRITE(6,304)
  304 FORMAT(//,20X,17HACCELERATION GAIN,/)
      WRITE(6,302) AGAIN
      G1=F(1)
      G2=F(2)
      G3=F(3)
      G4=F(4)
      VELKAN=GAIN/(G4+GAIN*TGAIN)
      WRITE(6,323) VELKAN
  323 FORMAT(//,20X,'COMPUTED VELOCITY CONSTANT IS =',F15.5,/)
      WRITE(6,601)
  601 FORMAT(/,1H1,/)
      C(1)=GAIN
      C(2)=G4+GAIN*TGAIN
      C(3)=G3+GAIN*AGAIN
      C(4)=G2
      C(5)=G1
      C(6)=1.0
      WRITE(6,105)
      WRITE(6,223) (C(I),I=1,6)
      DO 324 J=1,6
  324 W(I)=0.0
      DO 325 J=1,5
      ROOTR(J)=0.0
  325 ROOTI(J)=0.0
      CALL POLRT(C,W,5,ROOTR,ROOTI,IER)
      WRITE(6,107)
      WRITE(6,108)
      WRITE(6,116) (ROOTR(I),I=1,5)
      WRITE(6,117)
      WRITE(6,116) (ROOTI(I),I=1,5)
      WRITE(6,600)
       GO TO 999

C     THIS PROGRAM USES ROOT SPECIFICATIONS AND VELOCITY ERROR CONSTANT
C     FOR THIRD ORDER TACHOMETER AND ACCELEROMETER FEEDBACK SYSTEM

      REAL*8 A,B,PHI0,PHI1,PHI2,PHI3,PHI4,PHI5,P,D,E,ZETA,AMEGA,F,VELCON
     &,AMEGB
      DIMENSION B(3),A(3,3),P(4),D(2),E(2,2),R(3,3),C(5),W(5),ROOTR(5),R
     &OOTI(5),F(4)
```

```
999 READ(5,1) P(1),P(2)
    READ(5,1) P(3),P(4)
  1 FORMAT(2D15.5)
    READ(5,1) ZETA,AMEGA
    READ(5,1) VELCON,AMEGB
    PHI0=0.0
    PHI1=-1.0
    PHI2=2.0*ZETA
    PHI3=1.0-4.0*ZETA**2
    PHI4=-4.0*ZETA+8.0*ZETA**3
    PHI5=-1.0+12.0*ZETA**2-16.0*ZETA**4
300 WRITE(6,110)
110 FORMAT(1H1,//,20X,'INPUT DATA')
    WRITE(6,502)
502 FORMAT(20X,'**********',/)
    WRITE(6,321)P(1),P(2)
321 FORMAT(//,20X,'POLES ARE ZERO,',D14.5,',',D14.5,/,35X,D14.5,',',D1
   &4.5,/)
    WRITE(6,112) ZETA ,AMEGA
112 FORMAT(//,20X,'ROOT SPECIFICATIONS ARE ZETA=',D15.5,',',/,20X,'NAT
   &URAL FREQUENCY=',D15.5,/)
    WRITE(6,322) VELCON
322 FORMAT(//,20X,'VELOCITY CONSTANT IS',D15.5,/)
    WRITE(6,114)
114 FORMAT(//,20X,'NO BANDWIDTH SPECIFIED',/)
    A(1,1)=-1.0
    A(1,2)=VELCON
    A(1,3)=0.0
    A(2,1)=PHI0
    A(2,2)=AMEGA*PHI1
    A(2,3)=AMEGA**2*PHI2
    A(3,1)=-PHI1
    A(3,2)=AMEGA*PHI0
    A(3,3)=AMEGA**2*PHI1
    B(1)=-VELCON*P(1)*P(2)
    B(2)=-P(1)*P(2)*AMEGA*PHI1-(P(1)+P(2))*AMEGA**2*PHI2-AMEGA**3*PHI3
    B(3)=-P(1)*P(2)*AMEGA*PHI0-(P(1)+P(2))*AMEGA**2*PHI1-AMEGA**3*PHI2
    CALL DSIMQ(A,B,3,KS)
    GAIN=B(1)
    TGAIN=B(2)/B(1)
    AGAIN=B(3)/B(1)
    WRITE(6,115)
115 FORMAT(//,20X,'OUTPUT DATA',/)
    WRITE(6,501)
501 FORMAT(20X,'***********',/)
```

```
      WRITE(6,301)
  301 FORMAT(//,20X,12HFORWARD GAIN,/)
      WRITE(6,302) GAIN
  302 FORMAT(/,20X,E15.5,/)
      WRITE(6,303)
  303 FORMAT(//,20X,15HTACHOMETER GAIN,/)
      WRITE(6,302) TGAIN
      WRITE(6,304)
  304 FORMAT(//,20X,17HACCELERATION GAIN,/)
      WRITE(6,302) AGAIN
      VELKAN=GAIN/(P(1)*P(2)+GAIN*TGAIN)
       WRITE(6,323) VELKAN
  323 FORMAT(//,20X,'COMPUTED VELOCITY CONSTANT IS =',E15.5,/)
      C(1)=GAIN
      C(2)=P(1)*P(2)+GAIN*TGAIN
      C(3)=P(1)+P(2)+GAIN*AGAIN
      C(4)=1.0
      WRITE(6,105)
  105 FORMAT(//,20X,40HCOEFFICIENTS OF CHARACTERISTIC EQUATION,/,20X,'(A
     &SCENDING ORDER)',//)
      WRITE(6,223) (C(I),I=1,4)
  223 FORMAT(//,20X,4E15.5,/,20X,4E15.5,/)
      DO 324 J=1,4
  324 W(I)=0.0
      DO 325 J=1,3
      ROOTR(J)=0.0
  325 ROOTI(J)=0.0
      CALL POLRT(C,W,3,ROOTR,ROOTI,IER)
      WRITE(6,107)
  107 FORMAT(//,20X,21H THE SYSTEM ROOTS ARE,/)
      WRITE(6,108)
  108 FORMAT(////,20X,10H REAL PART,//)
      WRITE(6,116) (ROOTR(I),I=1,5)
  116 FORMAT(/,20X,8E15.5,/)
      WRITE(6,117)
  117 FORMAT(////,20X,15H IMAGINARY PART,//)
      WRITE(6,116) (ROOTI(I),I=1,5)
      WRITE(6,600)
  600 FORMAT(1H1,//,20X,'THE END',//)
       GO TO 999

C     THIS PROGRAM USES ROOT SECIFICATIONS AND VELOCITY ERROR FOR THIRD
C     ORDER SINGLE CASCADED COMPENSATION SYSTEM
```

```
      REAL*8 A,B,PHIO,PHI1,PHI2,PHI3,PHI4,PHI5,P,D,E,ZETA,AMEGA,F,VELCON
     &,AMEGB
      DIMENSION B(3),A(3,3),P(4),D(2),E(2,2),R(3,3),C(5),W(5),ROOTR(5),R
     &OOTI(5),F(4)
      READ(5,1) P(1),P(2)
    1 FORMAT(2D15.5)
      READ(5,1) ZETA,AMEGA
      READ(5,1) VELCON,AMEGB
  100 WRITE(6,110)
  110 FORMAT(1H1,//,20X,'INPUT DATA')
      WRITE(6,800)
  800 FORMAT(20X,'**********',/)
      WRITE(6,111) P(1),P(2)
  111 FORMAT(//,20X,'POLES ARE ZERO,',D15.5,',',D15.5,/)
      WRITE(6,112) ZETA,AMEGA
  112 FORMAT(//,20X,'ROOT SPECIFICATIONS ARE ZETA=',D15.5,',',/,20X,'NAT
     &URAL FREQUENCY=',D15.5,/)
      WRITE(6,113) VELCON
  113 FORMAT(//,20X,'VELOCITY CONSTANT IS',D15.5,//)
      WRITE(6,114)
  114 FORMAT(//,20X,'NO BANDWIDTH SPECIFIED',/)
      PHIO=0.0
      PHI1=-1.0
      PHI2=2.0*ZETA
      PHI3=1.0-4.0*ZETA**2
      PHI4=-4.0*ZETA+8.0*ZETA**3
      PHI5=-1.0+12.0*ZETA**2-16.0*ZETA**4
      A(1,1)=AMEGA*PHI1
      A(1,2)=P(1)*P(2)*AMEGA*PHI1+(P(1)+P(2))*AMEGA**2*PHI2+AMEGA**3*PHI
     &3
      A(1,3)=0.0
      A(2,1)=0.0
      A(2,2)=(P(1)+P(2))*AMEGA**2*PHI1+AMEGA**3*PHI2
      A(2,3)=-PHI1
      A(3,1)=0.0
      A(3,2)=-VELCON*P(1)*P(2)
      A(3,3)=1.0
      B(1)=-P(1)*P(2)*AMEGA**2*PHI2-(P(1)+P(2))*AMEGA**3*PHI3-AMEGA**4*P
     &HI4
      B(2)=-P(1)*P(2)*AMEGA**2*PHI1-(P(1)+P(2))*AMEGA**3*PHI2-AMEGA**4*P
     &HI3
      B(3)=0.0
      CALL DSIMQ(A,B,3,KS)
      WRITE(6,115)
```

```
  115 FORMAT(//,20X,'OUTPUT DATA',/)
      WRITE(6,801)
  801 FORMAT(20X,'***********',///)
      PO=B(2)
      SK=B(1)
      Z=B(3)/B(1)
      PRINT 3
    3 FORMAT(//,20X,'SYSTEM GAIN',//)
      PRINT 4,SK
    4 FORMAT(//,20X,E15.5,//)
      PRINT 40
   40 FORMAT(//,20X,'COMPENSATOR POLE',//)
      PRINT 4,PO
      PRINT 41
   41 FORMAT(//,20X,'COMPENSATOR ZERO',/)
      PRINT 4,Z
C
C     SPECIFICATION CHECK
C
      VELER=(Z*SK)/(3.5*PO)
      WRITE(6,504)
  504 FORMAT(//,1H1,/)
      PRINT 42
   42 FORMAT(//////,20X,'VELOCITY CONSTANT(COMPUTED)',//)
      PRINT 4,VELER
      DO 8 I=1,5
    8 W(I)=0.0
      C(1)=B(3)
      C(2)=3.5*PO+SK
      C(3)=7.5*PO+3.5
      C(4)=PO+7.5
      C(5)=1.0
      PRINT 9
    9 FORMAT(//,20X,'COEFFICIENTS OF CHARACTERISTIC EQUATION',/,20X,'(AS
     &CENDING ORDER)',//)
      PRINT 43,(C(I),I=1,5)
   43 FORMAT(//,20X,3E15.5,/,20X,3E15.5,//)
      DO 10 J=1,4
      ROOTR(J)=0.0
   10 ROOTI(J)=0.0
      CALL POLRT(C,W,4,ROOTR,ROOTI,IER)
      PRINT 11
   11 FORMAT(//,20X,'THE SYSTEM ROOTS ARE')
      PRINT 69
   69 FORMAT(////,20X,'REAL PART',//)
```

```
      PRINT 68,(ROOTR(I),I=1,4)
   68 FORMAT(//,20X,4E15.5,///)
      PRINT 70
   70 FORMAT(////,20X,'IMAGINARY PART',/)
      PRINT 68,(ROOTI(I),I=1,4)
      WRITE(6,504)

C     THIS PROGRAM USES ROOT SECIFICATIONS AND VELOCITY ERROR FOR THIRD
C     ORDER DOUBLE SECTION CASCADED COMPENSATION SYSTEM

      REAL*8 A,B,PHIO,PHI1,PHI2,PHI3,PHI4,PHI5,P,D,E,ZETA,AMEGA,F,VELCON
     &,AMEGB
      REAL*8 ZETA1,AMEGA1,BP,AP,X,ZW,Y,Z
      DIMENSION B(5),A(5,5),P(2),W(6),C(6),ROOTR(6),ROOTI(6)
  999 READ(5,1) P(1),P(2)
    1 FORMAT(2D15.5)
      READ(5,1) ZETA,AMEGA
      READ(5,1) VELCON,AMEGB
      PHIO=0.0
      PHI1=-1.0
      PHI2=2.0*ZETA
      PHI3=1.0-4.0*ZETA**2
      PHI4=-4.0*ZETA+8.0*ZETA**3
      PHI5=-1.0+12.0*ZETA**2-16.0*ZETA**4
  100 WRITE(6,110)
  110 FORMAT(//,20X,'INPUT DATA')
      WRITE(6,501)
  501 FORMAT(20X,'**********',/)
      WRITE(6,111) P(1),P(2)
  111 FORMAT(//,20X,'POLES ARE ZERO,',D15.5,',',D15.5,/)
      WRITE(6,112) ZETA,AMEGA
  112 FORMAT(//,20X,'ROOT SPECIFICATIONS ARE ZETA=',D15.5,',',/,20X,'NAT
     &URAL FREQUENCY=',D15.5,/)
      WRITE(6,113) VELCON
  113 FORMAT(//,20X,'VELOCITY CONSTANT IS=',D15.5,/)
      WRITE(6,114)
  114 FORMAT(//,20X,'NO BANDWIDTH SPECIFIED',/)
      AP=P(1)+P(2)
      BP=P(1)*P(2)
      A(1,1)=-PHI1
      A(1,2)=AP*AMEGA**2*PHI1+AMEGA**3*PHI2
      A(1,3)=0.0
      A(1,4)=AMEGA**2*PHI1
```

```
      A(1,5)=BP*AMEGA**2*PHI1+AP*AMEGA**3*PHI2+AMEGA**4*PHI3
      B(1)=-BP*AMEGA**3*PHI2-AP*AMEGA**4*PHI3-AMEGA**5*PHI4
      A(2,1)=0.0
      A(2,2)=BP*AMEGA*PHI1+AP*AMEGA**2*PHI2+AMEGA**3*PHI3
      A(2,3)=AMEGA*PHI1
      A(2,4)=AMEGA**2*PHI2
      A(2,5)=BP*AMEGA**2*PHI2+AP*AMEGA**3*PHI3+AMEGA**4*PHI4
      B(2)=-BP*AMEGA**3*PHI3-AP*AMEGA**4*PHI4-AMEGA**5*PHI5
      A(5,1)=-1.0
      A(5,2)=VELCON*BP
      A(5,3)=0.0
      A(5,4)=0.0
      A(5,5)=0.0
      B(5)=0.0
      READ(5,1) ZETA1,AMEGA1
      PHI0=0.0
      PHI1=-1.0
      PHI2=2.0*ZETA1
      PHI3=1.0-4.0*ZETA1**2
      PHI4=-4.0*ZETA1+8.0*ZETA1**3
      PHI5=-1.0+12.0*ZETA1**2-16.0*ZETA1**4
      A(3,1)=-PHI1
      A(3,2)=AP*AMEGA1**2*PHI1+AMEGA1**3*PHI2
      A(3,3)=0.0
      A(3,4)=AMEGA1**2*PHI1
      A(3,5)=BP*AMEGA1**2*PHI1+AP*AMEGA1**3*PHI2+AMEGA1**4*PHI3
      B(3)=-BP*AMEGA1**3*PHI2-AP*AMEGA1**4*PHI3-AMEGA1**5*PHI4
      A(4,1)=0.0
      A(4,2)=BP*AMEGA1*PHI1+AP*AMEGA1**2*PHI2+AMEGA1**3*PHI3
      A(4,3)=AMEGA1*PHI1
      A(4,4)=AMEGA1**2*PHI2
      A(4,5)=BP*AMEGA1**2*PHI2+AP*AMEGA1**3*PHI3+AMEGA1**4*PHI4
      B(4)=-BP*AMEGA1**3*PHI3-AP*AMEGA1**4*PHI4-AMEGA1**5*PHI5
      WRITE(6,112) ZETA1,AMEGA1
      CALL DSIMQ(A,B,5,KS)
      GAIN=B(4)
      Y=B(1)/B(4)
      Z=B(2)
      ZW=B(5)
      X=B(3)/B(4)
      WRITE(6,115)
  115 FORMAT(//,20X,'OUTPUT DATA')
      WRITE(6,502)
  502 FORMAT(20X,'***********',/)
      WRITE(6,101)
```

```
101 FORMAT(//,20X,12HFORWARD GAIN,/)
    WRITE(6,102) GAIN
102 FORMAT(/,20X,E15.5,/)
    DO 104 I=1,3
104 W(I)=0.0
    C(1)=Y
    C(2)=X
    C(3)=1.0
    DO 200 I=1,2
    ROOTR(I)=0.0
200 ROOTI(I)=0.0
    CALL POLRT(C,W,2,ROOTR,ROOTI,IER)
    WRITE(6,201)
201 FORMAT(//,20X,'FILTER ZEROS ARE',/)
    WRITE(6,108)
108 FORMAT(//,20X,10H REAL PART,//)
    WRITE(6,116) (ROOTR(I),I=1,2)
116 FORMAT(//,20X,4E15.5,/,20X,4E15.5,/)
    WRITE(6,117)
117 FORMAT(////,20X,'IMAGINARY PART',//)
    WRITE(6,116) (ROOTI(I),I=1,2)
    DO 302 I=1,3
302 W(I)=0.0
    C(1)=Z
    C(2)=ZW
    C(3)=1.0
    DO 303 I=1,2
    ROOTR(I)=0.0
303 ROOTI(I)=0.0
    CALL POLRT(C,W,2,ROOTR,ROOTI,IER)
    WRITE(6,503)
503 FORMAT(/,1H1,/)
    WRITE(6,304)
304 FORMAT(//,20X,'FILTER POLES ARE',//)
    WRITE(6,108)
    WRITE(6,116) (ROOTR(I),I=1,2)
    WRITE(6,117)
    WRITE(6,116) (ROOTI(I),I=1,2)
    DO 401 I=1,6
401 W(I)=0.0
    C(1)=B(1)
    C(2)=BP*B(2)+B(3)
    C(3)=BP*B(5)+AP*B(2)+B(4)
    C(4)=BP+B(2)+AP*B(5)
    C(5)=AP+B(5)
```

```
      C(6)=1.0
      WRITE(6,105)
  105 FORMAT(//,20X,'COEFFICIENTS OF CHARACTERISTIC EQUATION',/,20X,'(AS
     &CENDING ORDER)',//)
      WRITE(6,199) (C(I),I=1,6)
  199 FORMAT(//,20X,3E15.5,/,20X,3E15.5,//)
      DO 404 I=1,5
      ROOTR(I)=0.0
  404 ROOTI(I)=0.0
      CALL POLRT(C,W,5,ROOTR,ROOTI,IER)
      WRITE(6,107)
  107 FORMAT(//,20X,21H THE SYSTEM ROOTS ARE,/)
      WRITE(6,108)
      WRITE(6,116) (ROOTR(I),I=1,5)
      WRITE(6,117)
      WRITE(6,116) (ROOTI(I),I=1,5)


C     THIS PROGRAM USES BANDWIDTH AND VELOCITY ERROR FOR THIRD ORDER
C     TACHOMETER FEEDBACK SYSTEM


      DIMENSION W(4),C(4),ROOTR(4),ROOTI(4)
  999 READ(5,1) P1,P2
    1 FORMAT(2E15.5)
      READ(5,1)P3,P4
      READ(5,1) ZETA,AMEGA
      READ(5,1) VELCON,AMEGAB
      WRITE(6,2)
    2 FORMAT(1H1,//,20X,'INPUT DATA')
      WRITE(6,3)
    3 FORMAT(20X,'**********',//)
      WRITE(6,4) P1,P2
    4 FORMAT(//,20X,'POLES ARE ZERO,',E15.5,',',E15.5,/)
      WRITE(6,5)
    5 FORMAT(//,20X,' NO ROOT SPECIFICATION',/)
      WRITE(6,6) VELCON
    6 FORMAT(//,20X,' VELOCITY CONSTANT IS=',E15.5,/)
      WRITE(6,7)AMEGAB
    7 FORMAT(//,20X,'BANDWIDTH FREQUENCY IS=',E15.5,/)
      AP=-(P1+P2)*AMEGAB**2
      BP=P1*P2-AMEGAB**2
      CP=AP**2-(AMEGAB**2*BP**2)
      A=-1.0-AMEGAB**2/VELCON**2
      B=2.0*AP-2.0*AMEGAB**2*(BP-P1*P2)/VELCON
```

```
      C=CP+AMEGAB**2*P1*P2*(2.0*BP-P1*P2)
      GAIN1=0.5*(-B+SQRT(B**2-4.0*A*C))/A
      GAIN2=0.5*(-B-SQRT(B**2-4.0*A*C))/A
      TK1=(-P1*P2+GAIN1/VELCON)/GAIN1
      TK2=(-P1*P2+GAIN2/VELCON)/GAIN2
      WRITE(6,8)
    8 FORMAT(//,20X,'OUTPUT DATA')
      WRITE(6,9)
    9 FORMAT(20X,'***********',//)
       WRITE(6,10)GAIN1,TK1
   10 FORMAT(//,20X,'SYSTEM GAIN ONE=',E15.5,/,20X,'TACHOMETER GAIN ONE=
     &',E15.5,/)
      WRITE(6,11) GAIN2,TK2
   11 FORMAT(//,20X,'SYSTEM GAIN ONE=',E15.5,/,20X,'TACHOMETER GAIN TWO=
     &',E15.5,/)
      DO 100 I=1,4
      W(I)=0.0
      ROOTR(I)=0.0
  100 ROOTI(I)=0.0
      D(1)=GAIN2
      D(2)=P1*P2+GAIN2*TK2
      D(3)=P1+P2
      D(4)=1.0
      WRITE(6,101)
  101 FORMAT(//,20X,'COEFFICIENTS OF CHARACTERISTIC EQUATION',/,20X,'(AS
     &CENDING ORDER)',/)
      WRITE(6,102) (D(I),I=1,4)
  102 FORMAT(//,20X,4E15.5,/)
      CALL POLRT(D,W,3,ROOTR,ROOTI,IER)
      WRITE(6,103)
  103 FORMAT(//,20X,'THE SYSTEM ROOTS ARE')
      WRITE(6,104)
  104 FORMAT(////,20X,'REAL PART',/)
      WRITE(6,102) (ROOTR(I),I=1,3)
      WRITE(6,105)
  105 FORMAT(////,20X,'IMAGINARY PART',/)
      WRITE(6,102) (ROOTI(I),I=1,3)
      WRITE(6,106)
  106 FORMAT(/,1H1,/)
      GO TO 999

C     THIS PROGRAM USES BANDWIDTH AND VELOCITY ERROR FOR THIRD ORDER
C     ACCELEROMETER FEEDBACK SYSTEM
```

```
      DIMENSION W(4),D(4),ROOTR(4),ROOTI(4)
  999 READ(5,1) P1,P2
    1 FORMAT(2E15.5)
      READ(5,1)P3,P4
      READ(5,1) ZETA,AMEGA
      READ(5,1) VELCON,AMEGAB
      WRITE(6,2)
    2 FORMAT(1H1,//,20X,'INPUT DATA')
      WRITE(6,3)
    3 FORMAT(20X,'**********',//)
      WRITE(6,4) P1,P2
    4 FORMAT(//,20X,'POLES ARE ZERO,',E15.5,',',E15.5,/)
      WRITE(6,5)
    5 FORMAT(//,20X,'NO ROOT SPECIFICATION',/)
      WRITE(6,6) VELCON
    6 FORMAT(//,20X,'VELOCITY CONSTANT IS=',E15.5,/)
      WRITE(6,7)AMEGAB
    7 FORMAT(//,20X,'BANDWIDTH FREQUENCY IS=',E15.5,/)
      AP=P1*P2-AMEGAB**2
      BP=AMEGAB**2*(P1+P2)
      GAIN=P1*P2*VELCON
      A=AMEGAB**4*GAIN**2
      B=2.0*AMEGAB**2*(GAIN**2+BP*GAIN)
      C=BP**2-2.0*BP*GAIN-GAIN**2-AMEGAB**2*AP**2
      AK1=0.5*(-B+SQRT(B**2-4.0*A*C))/A
      AK2=0.5*(-B-SQRT(B**2-4.0*A*C))/A
      GAIN1=GAIN
      GAIN2=GAIN
      WRITE(6,8)
    8 FORMAT(//,20X,'OUTPUT DATA')
      WRITE(6,9)
    9 FORMAT(20X,'***********',//)
      WRITE(6,10)GAIN1,AK1
   10 FORMAT(//,20X,'SYSTEM GAIN ONE=',E15.5,/,20X,'ACCELERATION GAIN ON
     &E=',E15.5,/)
      WRITE(6,11) GAIN2,AK2
   11 FORMAT(//,20X,'SYSTEM GAIN TWO=',E15.5,/,20X,'ACCELERATION GAIN TW
     &O=',E15.5,/)
      DO 100 I=1,4
      W(I)=0.0
      ROOTR(I)=0.0
  100 ROOTI(I)=0.0
      D(1)=GAIN1
      D(2)=P1*P2
```

```
      D(3)=P1+P2+GAIN1*AK1
      D(4)=1.0
      WRITE(6,101)
  101 FORMAT(//,20X,'COEFFICIENTS OF CHARACTERISTIC EQUATION',/,20X,'(AS
     &CENDING ORDER)',/)
      WRITE(6,102) (D(I),I=1,4)
  102 FORMAT(//,20X,4E15.5,/)
      CALL POLRT(D,W,3,ROOTR,ROOTI,IER)
      WRITE(6,103)
  103 FORMAT(//,20X,'THE SYSTEM ROOTS ARE')
      WRITE(6,104)
  104 FORMAT(////,20X,'REAL PART',/)
      WRITE(6,102) (ROOTR(I),I=1,3)
      WRITE(6,105)
  105 FORMAT(////,20X,'IMAGINARY PART',/)
      WRITE(6,102) (ROOTI(I),I=1,3)
      WRITE(6,106)
  106 FORMAT(/,1H1,/)
      GO TO 999

C     THIS PROGRAM USES ROOT SPECIFICATION AND VELOCITY CONSTANT FOR
C     THIRD ORDER SINGLE SECTION CASCADED ACCELEROMETER FEEDBACK SYSTEM

      READ(5,1)P1,P2
    1 FORMAT(2E15.5)
      READ(5,1)ZETA,AMEGA
      READ(5,1)VELCON,SGAIN
      PHI0=0.0
      PHI1=-1.0
      PHI2=2.0*ZETA
      PHI3=1.0-4.0*ZETA**2
      PHI4=-4.0*ZETA+8.0*ZETA**3
      PHI5=-1.0+12.0*ZETA**2-16.0*ZETA**4
      WRITE(6,2)
    2 FORMAT(1H1,//,20X,'INPUT DATA')
      WRITE(6,3)
    3 FORMAT(20X,'**********',//)
      WRITE(6,4)P1,P2
    4 FORMAT(//,20X,'SYSTEM POLES ARE ZERO,',',',E15.5,',',E15.5,/)
      WRITE(6,5)ZETA,AMEGA
    5 FORMAT(//,20X,'ROOT SPECIFICATIONS ARE ZETA=',E15.5,',',/,20X,'NAT
     &URAL FREQUENCY=',E15.5,/)
      WRITE(6,6)VELCON
```

```
    6 FORMAT(//,20X,'VELOCITY CONSTANT IS =',E15.5,/)
      WRITE(6,7)
    7 FORMAT(//,20X,'NO BANDWIDTH SPECIFIED',/)
      A=VELCON*P1*P2
      B=-SGAIN
      C=(P1+P2)*AMEGA**2*PHI1+AMEGA**3*PHI2
      D=-SGAIN*PHI1
      E=SGAIN*AMEGA**2*PHI1
      F=SGAIN*AMEGA**3*PHI2
      G=-P1*P2*AMEGA**2*PHI1-(P1+P2)*AMEGA**3*PHI2-AMEGA**4*PHI3
      H=-SGAIN*AMEGA*PHI1-P1*P2*AMEGA**2*PHI2-(P1+P2)*AMEGA**3*PHI3-AMEG
     &A**4*PHI4
      R=AMEGA**3*PHI3+P1*P2*AMEGA*PHI1+(P1+P2)*AMEGA**2*PHI2
      S=SGAIN*AMEGA**2*PHI2
      T=SGAIN*AMEGA**3*PHI3
      W=D-C*B/A
      U=T*E-S*F
      V=T*W+R*B*F/A-S*G-H*E
      X=-R*B*G/A-H*W
      AK1=0.5*(-V+SQRT(V**2-4.0*U*X))/U
      AK2=0.5*(-V-SQRT(V**2-4.0*U*X))/U
      ZERO1=(G-F*AK1)/(W+E*AK1)
      ZERO2=(G-F*AK2)/(W+E*AK2)
      POLE2=-B*ZERO1/A
      POLE2=-B*ZERO2/A
      WRITE(6,8)
    8 FORMAT(//,20X,'OUTPUT DATA')
      WRITE(6,9)
    9 FORMAT(20X,'***********')
      WRITE(6,10)
   10 FORMAT(//,20X,'CHOICE ONE')
      WRITE(6,11)AK1,POLE1,ZERO1
   11 FORMAT(//,20X,'ACCELERATION GAIN=',E15.5,/,20X,'COMPENSATOR POLE='
     &,E15.5,/,20X,'COMPENSATOR ZERO=',E15.5,//)
      WRITE(6,12)
   12 FORMAT(//,20X,'CHOICE TWO')
      WRITE(6,13)AK2,POLE2,ZERO2
   13 FORMAT(//,20X,'ACCELERATION GAIN=',E15.5,/,20X,'COMPENSATOR POLE='
     &,E15.5,/,20X,'COMPENSATOR ZERO=',E15.5,//////)
      DO 100 I=1,5
      WW(I)=0.0
      ROOTR(I)=0.0
  100 ROOTI(I)=0.0
      CC(1)=500.0*ZERO2
      CC(2)=40.0*POLE2+500.0
```

```
      CC(3)=40.0+13.0*POLE2+AK2*SGAIN*ZERO2
      CC(4)=13.0+POLE2+SGAIN*AK2
      CC(5)=1.0
      WRITE(6,101)
  101 FORMAT(//,20X,'COEFFICIENTS OF CHARACTERISTIC EQUATION',/,20X,'(AS
     &CENDING ORDER)',/)
      WRITE(6,108) (CC(I),I=1,5)
  108 FORMAT(//,20X,4E15.5,/,20X,E15.5,/)
  102 FORMAT(//,20X,4E15.5,/,20X,E15.5,/)
      CALL POLRT(CC,WW,4,ROOTR,ROOTI,IER)
      WRITE(6,103)
  103 FORMAT(//,20X,'THE SYSTEM ROOTS ARE')
      WRITE(6,104)
  104 FORMAT(/,20X,'REAL PART',/)
      WRITE(6,102) (ROOTR(I),I=1,4)
      WRITE(6,105)
  105 FORMAT(/,20X,'IMAGINARY PART',/)
      WRITE(6,102) (ROOTI(I),I=1,4)
      WRITE(6,106)
  106 FORMAT(/,1H1,/)

C     THIS PROGRAM USES ROOT SPECIFICATION AND VELOCITY CONSTANT FOR
C     THIRD ORDER SINGLE SECTION CASCADED TACHOMETER FEEDBACK SYSTEM

      DIMENSION CC(5),WW(5),ROOTR(5),ROOTI(5)
      READ(5,1)P1,P2
    1 FORMAT(2E15.5)
      READ(5,1)ZETA,AMEGA
      READ(5,1)VELCON,SGAIN
      PHI0=0.0
      PHI1=-1.0
      PHI2=2.0*ZETA
      PHI3=1.0-4.0*ZETA**2
      PHI4=-4.0*ZETA+8.0*ZETA**3
      PHI5=-1.0+12.0*ZETA**2-16.0*ZETA**4
      WRITE(6,2)
    2 FORMAT(1H1,//,20X,'INPUT DATA')
      WRITE(6,3)
    3 FORMAT(20X,'**********',//)
      WRITE(6,4)P1,P2
    4 FORMAT(//,20X,'SYSTEM POLES ARE ZERO,',',',E15.5,',',E15.5,/)
      WRITE(6,5)ZETA,AMEGA
    5 FORMAT(//,20X,'ROOT SPECIFICATIONS ARE ZETA=',E15.5,',',/,20X,'NAT
```

```
     &URAL FREQUENCY=',E15.5,/)
      WRITE(6,6)VELCON
    6 FORMAT(//,20X,'VELOCITY CONSTANT IS =',E15.5,/)
      WRITE(6,7)
    7 FORMAT(//,20X,'NO BANDWIDTH SPECIFIED',/)
      A=VELCON*P1*P2
      B=VELCON*SGAIN
      C=-SGAIN
      D=AMEGA**2*PHI1*SGAIN
      E=(P1+P2)*AMEGA**2*PHI1+AMEGA**3*PHI2
      F=-SGAIN*PHI1
      G=-P1*P2*AMEGA**2*PHI1-(P1+P2)*AMEGA**3*PHI2-AMEGA**4*PHI3
      T=-SGAIN*AMEGA*PHI1-P1*P2*AMEGA**2*PHI2-(P1+P2)*AMEGA**3*PHI3-AMEG
     &A**4*PHI4
      H=AMEGA**2*PHI2*SGAIN
      S=AMEGA*SGAIN*PHI1
      R=P1*P2*AMEGA*PHI1+(P1+P2)*AMEGA**2*PHI2+AMEGA**3*PHI3
      W=S-R*B/A
      U=-E*B*H/A-D*W
      V=H*F+G*W+((R*D*C+T*E*B-H*E*C)/A)
      X=(T*E*C-G*R*C)/A-T*F
      TK1=0.5*(-V+SQRT(V**2-4.0*U*X))/U
      TK2=0.5*(-V-SQRT(V**2-4.0*U*X))/U
      ZERO1=(G-D*TK1)/(F-(E*C+E*B*TK1)/A)
      ZERO2=(G-D*TK2)/(F-(E*C+E*B*TK2)/A)
      POLE1=(-B*TK1*ZERO1-C*ZERO1)/A
      POLE2=(-B*TK2*ZERO2-C*ZERO2)/A
      WRITE(6,8)
    8 FORMAT(//,20X,'OUTPUT DATA')
      WRITE(6,9)
    9 FORMAT(20X,'***********')
      WRITE(6,10)
   10 FORMAT(//,20X,'CHOICE ONE')
      WRITE(6,11)TK1,POLE1,ZERO1
   11 FORMAT(//,20X,'TACHOMETER GAIN=',E15.5,/,20X,'COMPENSATOR POLE=',E
     &15.5,/,20X,'COMPENSATOR ZERO=',E15.5,//)
      WRITE(6,12)
   12 FORMAT(//,20X,'CHOICE TWO')
      WRITE(6,13)TK2,POLE2,ZERO2
   13 FORMAT(//,20X,'TACHOMETER GAIN=',E15.5,/,20X,'COMPENSATOR POLE=',E
     &15.5,/,20X,'COMPENSATOR ZERO=',E15.5,//////)
      DO 100 I=1,5
      WW(I)=0.0
      ROOTR(I)=0.0
  100 ROOTI(I)=0.0
```

```
      CC(1)=500.0*ZERO1
      CC(2)=40.0*POLE1+500.0*TK1*ZERO1+500.0
      CC(3)=40.0+13.0*POLE1+500.0*TK1
      CC(4)=13.0+POLE1
      CC(5)=1.0
      WRITE(6,101)
  101 FORMAT(//,20X,'COEFFICIENTS OF CHARACTERISTIC EQUATION',/,20X,'(AS
     &CENDING ORDER)')
      WRITE(6,108) (CC(I),I=1,5)
  108 FORMAT(/,20X,4E15.5,/,20X,E15.5)
  102 FORMAT(//,20X,4E15.5,/,20X,E15.5)
      CALL POLRT(CC,WW,4,ROOTR,ROOTI,IER)
      WRITE(6,103)
  103 FORMAT(//,20X,'THE SYSTEM ROOTS ARE')
      WRITE(6,104)
  104 FORMAT(/,20X,'REAL PART',/)
      WRITE(6,102) (ROOTR(I),I=1,4)
      WRITE(6,105)
  105 FORMAT(/,20X,'IMAGINARY PART',/)
      WRITE(6,102) (ROOTI(I),I=1,4)
      WRITE(6,106)
  106 FORMAT(/,1H1,/)
      GO TO 999
```

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	20
2.	Library Naval Postgraduate School Monterey, California 93940	2
3.	Naval Ship Systems Command (Code 2052) Department of the Navy Washington, D. C. 20360	1
4.	Prof. G. J. Thaler Department of Electrical Engineering Naval Postgraduate School Monterey, California 93940	5
5.	Prof. D. D. Siljak Department of Electrical Engineering University of Santa Clara Santa Clara, Calif. 95053	1
6.	Prof. Toyomi Ohta Electrical Department, Defense Academy Obaradai, Yokosuka, Japan	1
7.	LT H. H. Choe Division of Control Engineering University of Saskatchewan Saskatoon, Saskatchewan, Canada	1
8.	Dr. K. W. Han Institute of Electronics National Chiao-Tung University Hsinchu, Taiwan	1
9.	Dr. S. H. Kiu P. O. Box 1 Shih-yi-Fen Lungtan, Taiwan	1
10.	Dr. S. M. Seltzer D/51/-01 B/102 Lockheed Missiles & Space Co. P. O. Box 504 Sunnyvale, Calif. 94088	1

11. Mr. D. R. Towill 1
Welsh College of Advanced Technology
Cathays Park, Cardiff, Wales

12. Dr. A. G. Thompson 1
Department of Mechanical Engineering
University of Adelaide
Adelaide, Australia

13. Mr. James Cox 1
Bldg. 151, Dept. 55-35
Lockheed Missile & Space Co.
Sunnyvale, Calif. 94088

14. LT John A. Walker, Jr. 1
Philadelphia Naval Shipyard
Philadelphia, Pa. 19112

Unclassified
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

Computer Techniques for Implementing Linear Control System Design Using Algebraic Methods

4. DESCRIPTIVE NOTES (Type of report and, inclusive dates)

M.S. Thesis - Electrical Engineering - 1968

5. AUTHOR(S) (First name, middle initial, last name)

Walker, John A., Jr.
Lieutenant, United States Navy

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
June 1968	97	6

8a. CONTRACT OR GRANT NO. b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

[redacted] ~~is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval~~ [redacted]

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Naval Postgraduate School Monterey, California 93940

13. ABSTRACT

Design of linear control systems is examined using algebraic methods. Previous work indicates that the system describing equations are nonlinear. Various combinations of feedback and cascade compensated systems are analyzed using velocity constant, bandwidth and complex root specifications. Methods for linearizing these system equations are demonstrated and examples are solved using specific digital computer programs. Feasibility of a general computer program is discussed.

Unclassified
Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Algebraic Methods						
Linear Control Systems						

DD FORM 1 NOV 65 1473 (BACK)
S/N 0101-807-6821

Unclassified
Security Classification

A-31409

[redacted]